NAME......MARKING SCHEME...... INDEX NO....................
CANDIDATE'S SIGNATURE..................
DATE:....................

GATUNDU SOUTH FORM FOUR 2013 EVALUATION EXAM

121/2
MATHEMATICS
PAPER II
JULY/AUGUST 2013
TIME: 2 ½ HOURS

KENYA CERTIFICATE OF SECONDARY EDUCATION
FORM FOUR EVALUATION EXAMINATION

INSTRUCTIONS TO CANDIDATES

a) Write your name and index number in the space provided above
b) This paper consists of two sessions: Section I and section II.
c) Answer all the questions in the section I and **only five** questions from section II.
d) All answer and working must be written on the question paper in the space provided below each question.
e) Show all the steps in your calculations giving your answers at each stage in the spaces provided in each question.
f) Marks may be given for correct working even if the answer is wrong.
g) Non-programmable silent electronic calculators and KNEC Maths tables may be used except where stated otherwise.

FOR EXAMINERS' USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total

GRAND TOTAL	

SECTION I (50 Marks)
(Answer all questions in this section)

1. Find the speed of a car in kmh-1 that covers a distance of 3300m in 2 min 56 sec. (2 marks)

$\frac{3300M}{176 \sec}$

$18.75M/\sec$

$1hr = 60 \times 60$
$1hr = 3600$

$\frac{18.75 \times 3600}{1000}$

$\frac{67,500}{1000}$

=

$\frac{67,500}{1000}$

$67.5 km/h.$

2. Without using a mathematical tables or calculator express in surd form and simplify. (3 marks)

$$\frac{(\text{Sin } 60^o)^2 \tan 30^o}{\text{Cos } 45^o + \sin 45^o}$$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\text{Tan} 30 = \frac{1}{\sqrt{3}}$

$\cos 45 = \frac{1}{\sqrt{2}}$

$\sin 45 = \frac{1}{\sqrt{2}}$

$$\frac{\left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}$$

$$\frac{\frac{3}{4} \times \frac{1}{\sqrt{3}}}{\frac{2}{\sqrt{2}}} = \frac{3}{4\sqrt{3}} \div \frac{2}{\sqrt{2}}$$

$\frac{3}{4\sqrt{3}} \times \frac{\sqrt{2}}{2}$

$\frac{3\sqrt{2}}{8\sqrt{3}}$

3. Find the distance between the centre A of a circle whose equation is $2x^2 + 2y^2 + 6x + 10y + 7=0$ and the point B(-4, 1) (4 marks)

$\frac{}{2}$

$x^2 + 3x + y^2 + 5y + \frac{7}{5} = 0$

$x^2 + 3x + \left(\frac{3}{2}\right)^2 + y^2 + 5y + \left(\frac{25}{4}\right) = -\frac{7}{5}$

$\left(x + \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right) = -\frac{7}{5}$

$-\frac{7}{5} + \frac{9}{4} + \frac{25}{4} = \frac{-28+45+125}{20}$

$\frac{142}{20} = 7.1$

$A\left(-\frac{3}{2}, -\frac{5}{2}\right)$

$B(-4, 1)$

$\sqrt{\left(-4+\frac{3}{2}\right)^2 + \left(1+\frac{5}{2}\right)^2}$

$\sqrt{(-2.5)^2 + (3.5)^2}$

$\sqrt{6.25 + 12.25}$

$\sqrt{18.5}$ $\Rightarrow 4.30$

4. Nairobi and Kisumu lie on the same longitude. The latitude of Nairobi is 1^o12^1s and that of Kisumu is 5^o50^1N. Calculate the distance between them in nautical miles. (3 marks)

$1^o = 60'$
$x = 12'$

$\frac{12}{60}$

0.2^o

$1 = 60'$
$y = 50$

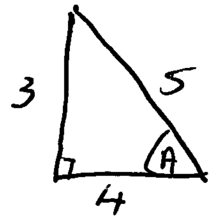
$\frac{50}{60} = 0.83$

$1.2 + 5.83$

$= 7.03^o$

$7.03 \times 60 = 421.8nM$

5. Given that tan A = $^3/_4$, find $\cos^2A + \sin^2A$. (3 marks)



$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$

$\cos^2 A + \sin^2 A =$

$\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2$

$\frac{16}{25} + \frac{9}{25} = \frac{25}{25} = \underline{\underline{1}}$

6. Make P the subject of the formula. (3 marks)

$$S = \sqrt{\frac{m}{2t}\left(n^2 - p^2\right)}$$

$s^2 = \frac{M}{2t}\left(n^2 - p^2\right)$

$\frac{2s^2t}{M} = n^2 - p^2$

$\frac{2s^2t}{M} - n^2 = -p^2$

$p^2 = n^2 - \frac{2s^2t}{M}.$

$p = \sqrt{n^2 - \frac{2s^2t}{M}}.$

$p = \pm\sqrt{n^2 - \frac{2ts^2}{M}}.$

7. Mweru bought probox at ksh. 800,000 when new. He sold it to Kamau after five years for ksh.350,000, but the rate of the probox was depreciating semi-annually. Calculate the rate of depreciation. (3 marks)

$A = P\left(1 - \frac{r}{100}\right)^n.$

$350{,}000 = 800{,}000\left(1 - \frac{R}{200}\right)^{10}$

$\sqrt[10]{0.4375} = \sqrt[10]{\left(1 - \frac{r}{200}\right)^{10}}$

$0.9207 = 1 - \frac{r}{200}$

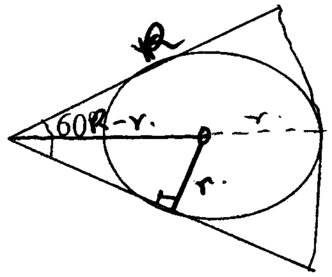
$\frac{r}{200} = 1 - 0.9207$

$\frac{r}{200} = 0.0793$

$r = 0.0793 \times 200$

$r = \underline{\underline{15.86\%}}$

8. Using the below figure showing a circle of radius r inscribed in a sector of a circle of radius R. Find:



a) the ratio of R : r (2 marks)

$\sin 30 = \frac{r}{R-r}$

$0.5 = \frac{r}{R-r}$

$0.5R - 0.5r = r.$

$0.5R = r + 0.5r.$

$\frac{0.5R}{0.5} = \frac{1.5r}{0.5}.$

$0.1R = 0.3r.$

$\underline{\underline{3:1}}$

b) the ratio of the area of the circle to the area of the sector.

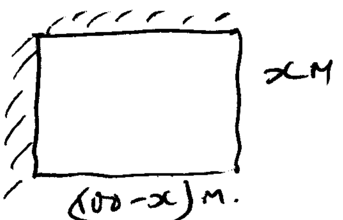
$$\frac{\frac{60}{360}\times(3r)^2\times\pi}{\pi r^2}$$

$$\frac{\frac{1}{6}\times 9r^2\pi}{\pi r^2} \quad \frac{3}{2}$$

$\frac{A \text{ of cir}}{A \text{ sec}}$ $\frac{3}{2}$

3 : 2

9. A farmer has 100m of metal railing with which to form two adjacent sides of a rectangular enclosing the other two sides being existing walls of the yards meeting at right angles. What dimensions will give him maximum area. (3 marks)



$A = 100x - x^2$

$\frac{dA}{dx} = 100 - 2x = 0$

$100 = 2x$

$x = 50$

$\dim = 50 \times 50$

10. Expand and simplify the binomial, hence find the constant term. (3 marks)

$$\left(x - \frac{4}{x}\right)^6$$

1, 6, 15, 20, 15, 6, 1

$x^6 \cdot x^5\left(\frac{-4}{x}\right) \cdot x^4\left(\frac{-4}{x}\right)^2 \cdot x^3\left(\frac{-4}{x}\right)^3 \cdot x^2\left(\frac{-4}{x}\right)^4 \cdot x^1\left(\frac{-4}{x}\right)^5 \cdot \left(\frac{-4}{x}\right)^6$

$x^6 - 24x^4 + 240x^2 - 1280 + \frac{3840}{x^2} - \frac{6144}{x^4} + \frac{4096}{x^6}$

constant -1280

11. Given that matrices $P = \begin{pmatrix} 1 & -3 \\ -2 & 5 \end{pmatrix}$, $Q = \begin{pmatrix} -4 & 0 \\ 1 & -3 \end{pmatrix}$ and R = PQ, Find the inverse of R. (3 marks)

$$\begin{pmatrix} 1 & -3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} -4 & 0 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} -7 & 9 \\ 13 & -15 \end{pmatrix}$$

$R \begin{pmatrix} -7 & 9 \\ 13 & -15 \end{pmatrix}$

det $(105 - 117)$

det $= -12$

$$\frac{-1}{12}\begin{pmatrix} -15 & -9 \\ -13 & -7 \end{pmatrix}$$

$$\begin{pmatrix} \frac{15}{12} & \frac{9}{12} \\ \frac{13}{12} & \frac{7}{12} \end{pmatrix}$$

12. Without using mathematical tables or calculator, solve for P in the equation below. (3 marks)

$\log_{(2x-7)}25 = 2$

13. Triangle ABC has co-ordinates A(2, 0), B2, 4) and C(4, 4). It is mapped onto $A^{11}(0,-2)$, $B^{II}(-4, -10)$ and $C^{II}(-4, -12)$ by two successive transformation $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ followed by $T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ .Find M. (4 marks)

$\cancel{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -4 & -4 \\ -2 & -10 & -12 \end{pmatrix} = \begin{pmatrix} 2 & 10 & 12 \\ 0 & 4 & 4 \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 2 & 2 & 4 \\ 0 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 10 & 12 \\ 0 & 4 & 4 \end{pmatrix}$

$2a + 0b = 2$ | $2a + 4b = 10$

$2c + 0d = 0$ | $2c + 4d = 4$

$a = 1$ $c = 0$

$b = 2$

$d = 1$

$M \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

14. Determine the interquartile range for the following set of numbers. (3 marks)

4, 9, 5, 4, 7, 6, 2, 1, 6, 7, 8

1, 2, 4, 4, 5, 6, 6, 7, 7, 8, 9

$Q_1 \quad \frac{11+1}{4} = 3^{rd}$

$= 4$

$\frac{12}{4} \times 3 = 9^{th}$

$= 7$

Range $= 7 - 4 = 3$

$= 3$

15. Given that $a = 5\underset{\sim}{i} + 4\underset{\sim}{j} + \mathbf{k}$ and $b = 8\underset{\sim}{i} - 5\underset{\sim}{j} - 6\underset{\sim}{k}$, find:

a) $\mathbf{p} = 2\mathbf{a} + \mathbf{b}$ (2 marks)

$p = 2\begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} + b\begin{pmatrix} 8 \\ -5 \\ -6 \end{pmatrix}$

$\begin{pmatrix} 10 \\ 8 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ -5 \\ -6 \end{pmatrix} = \begin{pmatrix} 18 \\ 3 \\ -4 \end{pmatrix}$

$18i + 3j - 4k$

b) $/\underset{\sim}{\mathbf{p}}/$ (1 mark)

$|p| = \sqrt{18^2 + 3^2 + 4^2}$

$\sqrt{324 + 9 + 16}$

$\sqrt{349}$

18.68 units.

16. Using mid-ordinates rule estimate the area under the curve $y=2x^2-8x+10$ and lines $y=0$, $x=0$ and $x=5$. (3 marks)

x	0.5	1.5	2.5	3.5	4.5
$y=2x^2-8x+10$	6.5	2.5	2.5	6.5	14.5

$A = 1(6.5+2.5+2.5+6.5+14.5)$

32.5 sq units

SECTION II (50 Marks)

ANSWER ANY FIVE QUESTIONS FROM THIS SECTION

17. The rate R for announcing t times through the radio during campaign for "Jubilee" varies partly as t and partly as the square of t. To announce 200 times the rate was ksh.3,810,000 and to announce 80 times the rate was ksh.612,000. Find:

a) the constant of proportionality in announcing and hence write the law connecting R and t. (6 marks)

$R = at + bt^2$

$3810{,}000 = 200a + 40000b$

$612\,000 = 80a + 6400b$

$7620{,}000 = 400a + 80{,}000b$

$3060000 = 400a + 32{,}000b$

$4{,}560{,}000 = 48{,}000b$

$b = 95$

$612\,000 = 80a + 608{,}000$

$4000 = 80a.$

$a = 50$

$R = 50t + 95t^2$

b) the rate of announcing 500 times. (2 marks)

$R = 50 \times 500 + 95 \times 500$

$R = 25{,}000 + 23{,}750{,}000$

$R = 23{,}775{,}000$

c) the number of times if the rate of announcement was ksh.214,500. (2 marks)

$214{,}500 = 50t + 95t^2$

$95t^2 + 50t - 214500$

$19t^2 + 10t - 429{,}000 = 0$

$t = \dfrac{-10 \pm \sqrt{100 + 32604100}}{38}$

$\dfrac{-10 \pm \sqrt{32604100}}{38}$

$t = \dfrac{-10 \pm 5710}{38}$

$t = \dfrac{5700}{38}$

$t = 150$

18. In a research carried out to try a drug on treatment for East cost fever a sample of 45 cows was diagnosed to have the disease: 25 cows were treated with the drug and the rest were not. P of treated & die is 4/15 & not treated and will die is 13/15

a) Calculate the probability that a cow picked at random is:

(i) treated with the drug and will die. (3 mark)

$\frac{25}{45} \Rightarrow \frac{5}{9}$

$\frac{5}{9} \times \frac{4}{15} = \frac{4}{27}$

(ii) a cow will not die. (3 mark)

$\left(\frac{5}{9} \times \frac{11}{15}\right) + \left(\frac{4}{9} \times \frac{2}{15}\right)$

$\frac{11}{27} + \frac{8}{135} = \frac{55+8}{135}$

$\frac{63}{135}$

b) The research found that if a cow is treated, the probability of it dying is 4/5 while if not treated the probability t~~hat a cow picked at random from 45 cows is:~~ of it dying is 13/15

(i) treated with the drug and will die. (3 marks)

(ii) a cow will not die (3 marks)

(iii) not treated with the drug will not die. (2 marks)

$\frac{4}{9} \times \frac{2}{15} = \frac{8}{135}$

19. By filling the table below:

a) draw the graph of $y=x^2 + 4x + 1$ and $y=x^3 - 3x + 1$ on the same axes for $-4 \leq x \leq 3$. (5 marks)

x	-4	-3	-2	-1	0	1	2	3
$y=x^2+4x+1$	1	-2	-3	-2	1	6	13	22
$y=x^3-3x+1$	-51	-17	-1	3	1	-1	3	19

b) Use the graph to solve

$x^3 - 3x + 1 = x^2 + 4x + 1$ (1 mark)

c) **Use integration to find the area of the region bounded by the curve's the x axis and x = 3.**
(4 marks)

$$\int_1^3 (x^2+4x+1)\,dx - \int_1^{1.6} (x^3-3x+1)\,dx - \int_{1.6}^3 (x^3-3x+1)\,dx$$

$$\left[\frac{x^3}{3}+2x^2+x\right]_1^3 - \left(\frac{x^4}{4}-\frac{3}{2}x+x\right)_1^{1.6} - \left[\frac{x^4}{4}-2x+x\right]_{1.6}^3$$

c) Construct altitude of triangle BCD from B meeting AC at Y. (2 marks)

d) Measure XY. (2 marks)

21. The first, third and sixth terms of an Arithmetic progression (AP) correspond to the first three consecutive terms of an increasing Geometric Progression (G.P). The first term of each progression is 16, the common difference of AP is d and the common ratio of the GP is r.

(a) (i) Write two equations involving d and r. (2 marks)

$a, a+2d, a+5d$

$\frac{a+2d}{a} = \frac{a+5d}{a+2d} = r$

$a^2+4ad+4d^2 = a^2+5ad$

$4d^2 = ad$

$4d = a$

$\frac{a+2d}{a} = r.$

(ii) Find the value of d and r (4 marks)

$4d = 16$

$d = 4$

$16 + 8 = 16r.$

$24 = 16r$

$r = \frac{24}{16}$

$r = 1.5$

(b) Find the sum of the first 20 terms:

(i) the Arithmetic progression (AP) (2 marks)

$S_{20} = \frac{20}{2}\{32 + 19 \times 4)$

$10(108)$

$S_{10} = 1080$

(ii) the Geometric Progression (GP) (2 marks)

$S_n = \frac{a(r^{20}-1)}{1.5-1}$

$S_{20} = \frac{16(1.5^{20}-1)}{0.5}$

$\frac{16(3,325.3}{0.5}$

$106,408.22$

22. Complete the table below for $y=2\sin\Theta^\circ + 1$ and $y=3\cos(\Theta + 30)^\circ$ (2 marks)

Θ°	0°	30°	60°	90°	120°	150°	180°	210°	240°
$2\sin\Theta^\circ + 1$	1	2	2.7	3	2.7	2	1.0	0	-0.7
$3\cos(\Theta + 30)^\circ$	2.6	1.5	0	-1.5	-2.6	-3	-2.6	-1.5	0

a) Draw the graph of $y=2\sin\Theta^\circ + 1$ and $y=3\cos(\Theta + 30)^\circ$ for $0^\circ<\Theta<240^\circ$ on the same axes. (4 marks)

b) (i) Find the values of Θ for which

$2\sin\Theta + 1 = 3\cos(\Theta + 30)^\circ$ (2 marks)

$\Theta = 24 \pm 1^\circ$

or

$\Theta = 231 \pm 1^\circ$

(ii)State the period and amplitude for

$y=3\cos(\Theta + 30)^\circ$ (2 marks)

Amplitude = 3

Period = 360°

23. A school basketball team played 84 games in one year and recorded the number of points scored in each year as shown below.

Number of points	Number of games	cf
7 – 12	2	2
13 – 18	3	5
19 – 24	5	10
25 – 30	12	22
31 – 36	25	47

37 – 42	20	67
43 – 48	11	78
49 – 54	3	81
55 - 60	3	84

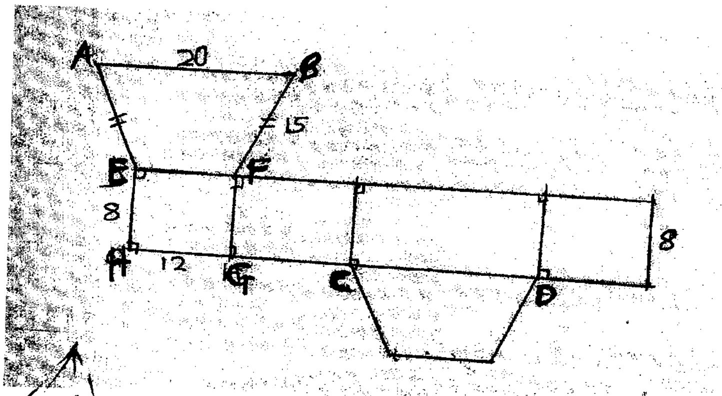
Draw the cumulative frequency curve and use it to estimate. (3 marks)

a) the median (2 marks)

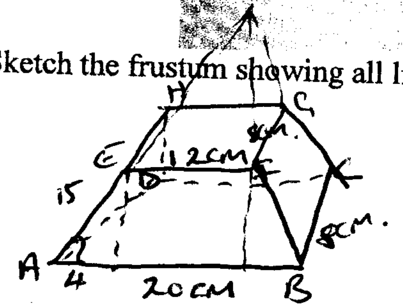
b) the quartile deviation (2 marks)

c) the number of games in which the team scored between 21 and 46 points inclusive. (3 marks)

24. The figure below (not drawn to scale) shows a net of a rectangular based of a solid prism where AE=BF=15cm, AB=20cm HG=EF=12cm and EH=FG=8cm. (All measurements are in centimeters)



a) Sketch the frustum showing all lines and labeled. (3 marks)



b) Calculate the volume of the frustum. (5 marks)

$\frac{20}{12} = \frac{L+15}{L}$

$20L = 12L + \cancel{300}\ 180$

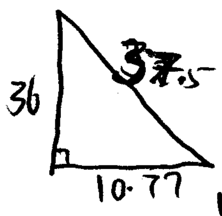
$8L = \cancel{180}$

$L = \frac{\cancel{180}}{8}\ 22.5$

$\cancel{L = 37.5}$

$\sqrt{464} = 21.54$

10.77cm.



$h = \sqrt{1406.25 - 116}$

c) Calculate the angle between planes ABCD and EFCD. (2 marks)

$\frac{1}{3} \times 20 \times 8 \times \frac{36}{12}$

$V = 1920cm^3$

$\frac{20}{12} \Rightarrow 5:3$ L.S.F.

A.V.S.F $= \frac{125}{27} = \frac{1920}{x}$

$x = \frac{1920 \times 27}{125}$

$x = \frac{51,840}{125}$

$x = 414.72cm^2$

$\Rightarrow 1920 - 414.72 = 1505.28cm^3$

(c) $\cos\theta = \frac{4}{15}$

$\cos\theta = 0.26$

$74.53°$