

FORM A MATHEMATICS MID-TERM EXAM 2013.

SECTION I (50 marks)

Answer all the questions in this section in the space provided.

1. Use logarithms to evaluate to 3 significant figures.

(4 marks)

$$\frac{(0.5241)^2 \times 83.59}{\sqrt[3]{0.3563}}$$

2. Find the value of C that will make the expression $16x^2 + 4x + c + 7$ a perfect square.

(3 marks)

3. A rectangular carpet was measured and found to have a length of 4.1 m and a breadth of 2.20 m. Determine the percentage error in the area of the carpet.

(3 marks)

4. Expand $(3 + \frac{1}{8}x)^4$ hence evaluate $(3.025)^4$ to 3 decimal places.

(4 marks)

5. Without using tables simplify $\frac{\log 4/9}{\log 27/8} + 1$

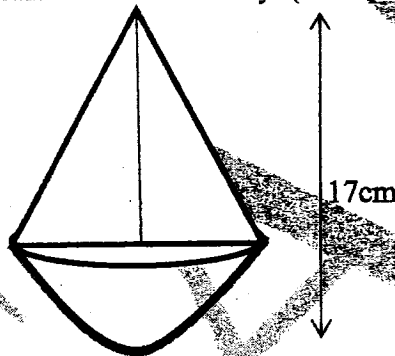
(3 marks)

6. Solve the inequality $3 - 2x < x \leq \frac{2x+5}{3}$ and show the solution on a number line. (3 marks)

7. Make S the subject of the formula in $P = \frac{r(1-s)}{s+1}$

(3 marks)

8. The figure below shows a toy which consists of a conical top and a hemispherical base. The hemispherical base has a radius of 5 cm and the total height of the toy is 17 cm. Calculate the surface area of the toy. (take $\pi = 3.142$) (3 marks)



9. A variable M varies directly as H and the square of r. Find the percentage change in M if r is decreased by 15 % and h is increased by 12 %.

(3 marks)

10. Expand $(3x - 7)(2x + 7)$

(3 marks)

11. Given that $\frac{x}{(x+2y)^3} = \frac{3}{8}$, find the ratio $x:y$.

(3 marks)

12. Without using tables or a calculator, evaluate $\frac{1}{(1+3)^2} + \frac{1}{(1-3)^2}$

(3 marks)

13. Given that $\begin{bmatrix} x-1 & x+1 \\ 3x & x \end{bmatrix}$ is a singular matrix, find the possible values of x .

(3 marks)

14. Mr. Saidi keeps turkeys and chickens. The number of turkeys exceeds the number of chickens by 6. During an outbreak of a disease, $\frac{1}{4}$ of the chicken and $\frac{1}{3}$ of the turkeys died. If he lost total of 30 birds, how many birds did he have altogether? (3 marks)

15. The base of a rectangular tank is 3.2 m by 2.8 m. Its height is 2.4 m. It contains water to a depth of 1.8 m. Calculate the volume inside the tank that is not in contact with water. (2 marks)

16. Given the column vector $p = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $q = \begin{pmatrix} 12 \\ -8 \end{pmatrix}$ and $r = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$ and that $a = 2p - \frac{3}{4}q + \frac{2}{3}r$, express a as a column vector and hence find its magnitude (4 marks)

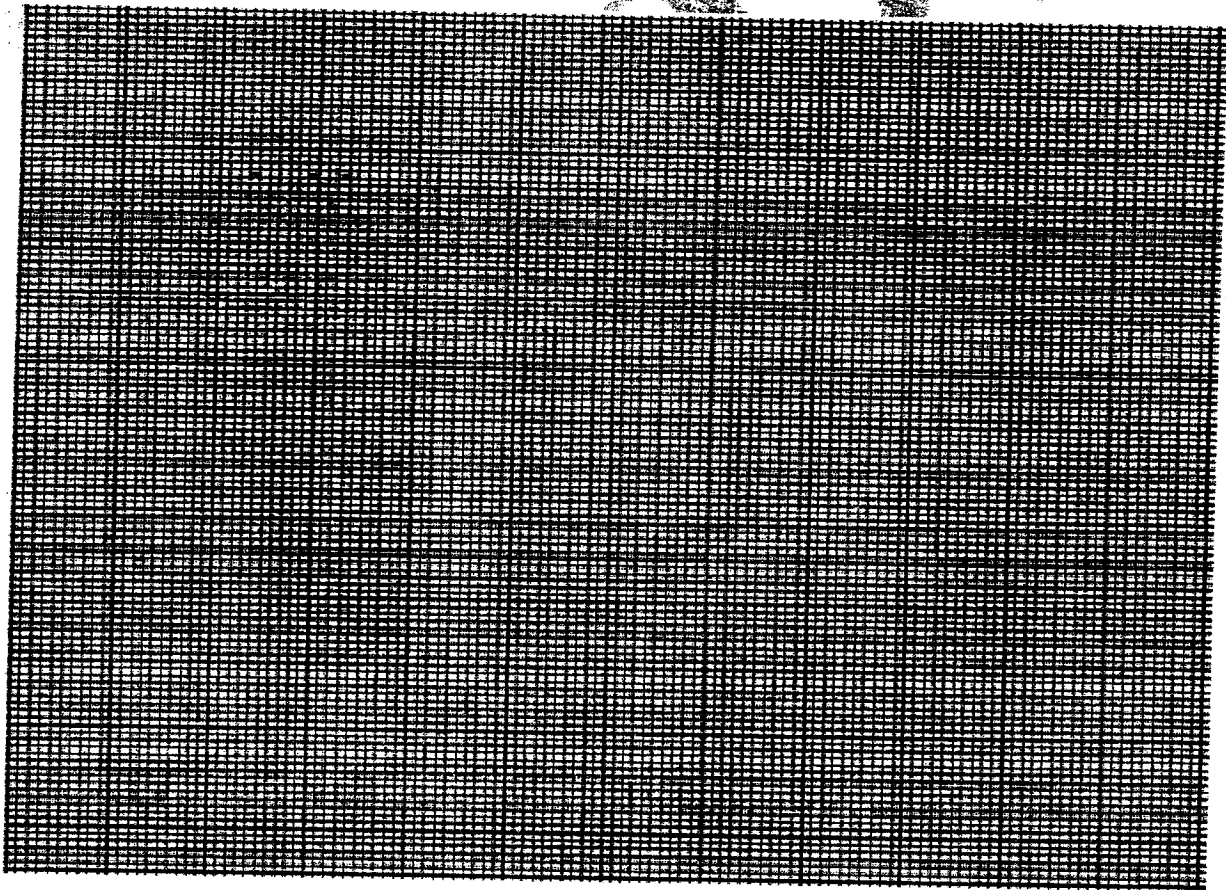
SECTION II (50 marks)

Answer any five questions in this section in the space provided.

17. The table below shows the life time of a random sample of 200 bulbs taken from a production line of a particular manufacturer.

Lifetime (hrs)	700-719	720-729	730-739	740-749	750-754	755-759	760-764	765-769	770-779	780-799
Number of bulbs	10	14	16	21	35	41	38	15	7	3
C. F	10	24	40	61	96	137	175	190	197	200

(a). Draw a cumulative frequency curve for the above data. (4 marks)



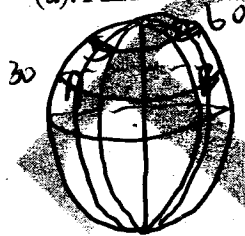
(b). Use the curve in (a) above to determine:
 (i) The median (1 mark)

(ii) The interquartile range

(3 marks)

(iii) The percentage number of bulbs that were found to be of good quality if the expected minimum lifetime was supposed to be 735 hours. (2 marks)

18. A ($30^{\circ}\text{N}, 20^{\circ}\text{W}$), B ($30^{\circ}\text{N}, 40^{\circ}\text{E}$), C ($60^{\circ}\text{N}, y^{\circ}\text{E}$) and D ($x^{\circ}\text{N}, z^{\circ}\text{W}$) are on the surface of the earth. C is due north of B and D is due west of C and also due north of A.
 (a) Find the values of x, y and z. (3 marks)



$$\begin{aligned} x &= 60^{\circ}\text{N} \\ z &= 20^{\circ}\text{W} \\ y &= 40^{\circ}\text{E} \end{aligned}$$

(b) If all distances are measured along the circle of latitudes and/or meridians, find the distances of C from A in m, along the routes ABC and ADC. (4 marks)

$$\begin{aligned} A & (30^{\circ}\text{N}, 20^{\circ}\text{W}) \\ B & (30^{\circ}\text{N}, 40^{\circ}\text{E}) \\ C & (60^{\circ}\text{N}, 40^{\circ}\text{E}) \end{aligned}$$

$$60 \times 60 = \underline{\underline{3600 \text{ km}}}$$

$$\begin{aligned} 30 \times 60 \cos 40 & \\ 1800 \times 0.7660 & \\ 1,378.9 & \\ \underline{3600} & \\ 4978.9 & \end{aligned}$$

- (c). Two aeroplanes start from A to fly to C. One flies along the route ABC at a speed of 400 knots and the other flies along the route ADC at a speed of 300 knots which plane reached C earlier and by what time. (3 marks)

19. A student in a maths club has six identical number cards 1-6 and five letters cards a,b,c,d,e. The students ask some of the members to pick one number card and one letter card then records the outcome.

(a)

(i). Prepare his/her probability space

(2 marks)

c	1	2	3	4	5	6
a	a1	a2	a3	a4	a5	a6
b	b1	b2	b3	b4	b5	b6
c	c1	c2	c3	c4	c5	c6
d	d1	d2	d3	d4	d5	d6

(ii). Find the probability that the letter C was a constant

(2 marks)

(iii). The number card was an even number

(2 marks)

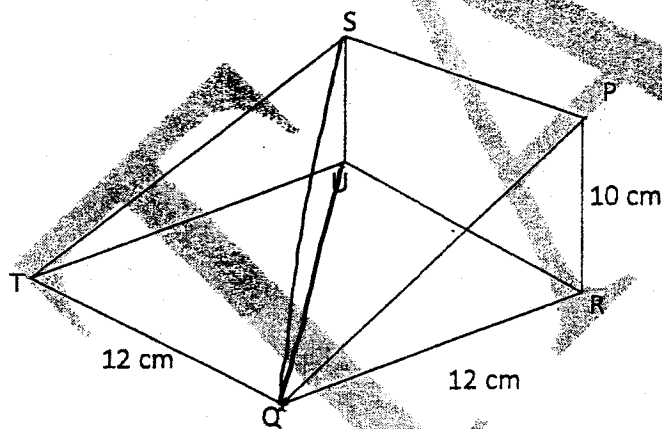
(iv). The letter card was a vowel and the number card was a prime number (2 marks)

(b) The table shows marks scored in a mathematics test by 20 candidates.

MARKS	2	4	5	6	7
NUMBER OF CANDIDATES	1	4	8	5	2

A candidate was chosen at random. Find the probability that he scored more than the median (2 marks)

20. The figure below shows a metallic wedge with $QR=12\text{cm}$, $QT=12\text{cm}$ and $PR=10\text{cm}$ plane $PRVS$ is vertical to plane $QRVT$



Calculate;

(i) The length of the diagonal QS.

(3 marks)

$$144 + 144 = \sqrt{288}$$

$$= 16.97\text{cm}$$

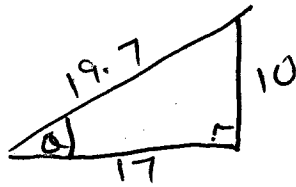
$$SQ = \underline{\underline{19.7\text{cm}}}$$

$$SQ = \sqrt{10^2 + 16.97^2}$$

$$SQ = \sqrt{388}$$

(ii) The angle that QS makes with the plane QRUT.

(3 marks)



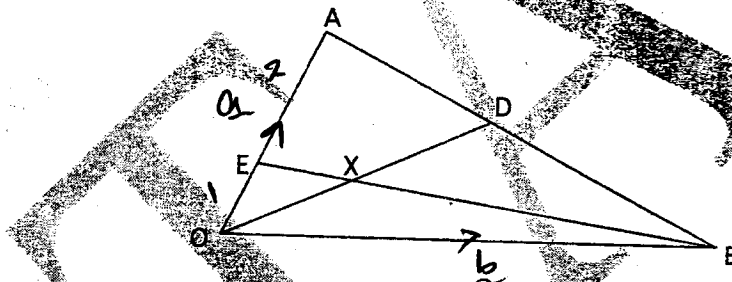
$$\begin{aligned} \cos \alpha &= \frac{17}{19.7} \\ \sin \alpha &= \frac{10}{19.7} \\ \sin \alpha &= 0.5076 \\ \alpha &= \underline{\underline{30.5^\circ}} \end{aligned}$$

(iii) The mass in kgs of the wedge if the density of the metal is 15g/cm^3 . (4 marks)

$$\begin{aligned} \frac{1}{2} \times 10 \times 12 &= 60 \times 12 \\ &= 720\text{cm}^3 \\ \rho &= \frac{M}{V} \end{aligned}$$

$$\begin{aligned} \frac{15\text{g}}{\text{cm}^3} \times 720\text{cm}^3 &= 10800\text{g} \\ &= \underline{\underline{10.8\text{kg}}} \end{aligned}$$

21. Triangle OAB is such that the position vector of A is \mathbf{a} and that of B is \mathbf{b} . Given that $4\mathbf{AD} = \mathbf{AB}$ and E divides line OA in the ratio 1:2. X is the point of intersection of OD and BE.



(a). Express in terms of \mathbf{a} and \mathbf{b} the vectors in their simplest form.

(i) OD

(2 marks)

(ii) DE

(2 marks)

$$n = -1$$

$$n = -0.3010$$

$$\frac{-0.07058}{-0.3010}$$

$$n = \frac{\log 0.85}{\log 0.5}$$

$$\log 0.5 = n \log 0.85$$

$$0.5 = (0.85)^n$$

$$55,000 = 110,000 (0.85)^n$$

(b) The number of years it will take for the value of the machine to depreciate to 55 000

$$n = 4.23$$

(4 marks)

(a) The value of the machine after 3 years.

$$A = 110,000 (1 - \frac{15}{100})^3$$

$$A = 110,000 (0.85)^3$$

$$A = 110,000 \times 0.614125$$

$$A = 67,553.75$$

22. A farmer bought a machine at ksh 110 000. If the depreciation of the machine is at a rate of 15% p.a. Calculate:

(i) Find OX in terms of a and b only

(b) If $OX = mOD$ and $BX = nBE$.

(6 marks)

(c) The value of the machine after 5 years (to the nearest cent)

(3 marks)

$$110,000(0.85)^5$$
$$110,000 \times 0.4437$$
$$\approx \underline{\underline{48,807.60}}$$

23. Find the equation of a straight line:

(a) Passing through the points (3,2) and (-3,6) giving your answer in the form $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are constants

(4 marks)

$$\frac{6-2}{-3-3} = \frac{4}{-6}$$

$$m = -\frac{2}{3}$$

$$\frac{y-2}{x-3} = -\frac{2}{3}$$

$$y-2 = -\frac{2}{3}(x-3)$$

$$y-2 = -\frac{2}{3}x + \frac{6}{3}$$

$$y = -\frac{2}{3}x + 4$$

$$y + \frac{2}{3}x = 4$$

$$\frac{y}{4} + \frac{x}{6} = 1$$

(b) State the coordinates of point A and B at which the line in (a) above crosses the x-axis and y-axis respectively

(2 marks)

$$A(x, 0)$$

$$\frac{2}{3}x = 4$$

$$x = 4 \times \frac{3}{2}$$

$$x = 6 \quad A(6, 0)$$

$$B(0, y)$$

$$y = 4$$

$$B(0, 4)$$

- (c) Using the information in (a) and (b) above, find the area of triangle AOB where O is the origin (2 marks)

- (d) Find the acute angle the line in (a) above makes with the axis (2 marks)

$$y + \frac{2}{3}x = 4$$

$$y = -\frac{2}{3}x + 4$$

$$m = -\frac{2}{3}$$

$$\tan \theta = -\frac{2}{3}$$

$$\tan \theta = -0.6667$$

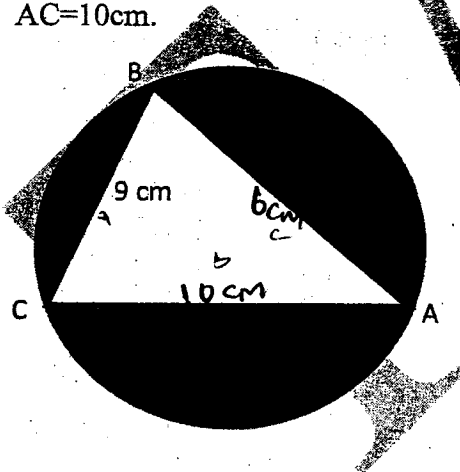
$$\theta = 33.7^\circ$$

$$180 - 33.7^\circ$$

$$146.3^\circ$$

$$33.7^\circ$$

24. The figure below shows a triangle ABC inscribed in a circle, AB=6cm, BC=9cm and AC=10cm.



Calculate to 1 d.p.

- (a) The radius of the circle.

$$9^2 = 10^2 + 6^2 - 120 \cos A$$

$$-55 = -120 \cos A$$

$$0.45833 = \cos A$$

$$A = 117.28^\circ$$

$$\frac{9}{\sin 117.28^\circ} = 2R$$

$$\frac{9}{0.888} = 2R$$

$$R = \frac{9}{2 \times 0.888}$$

$$R = \frac{9}{1.7776}$$

$$R = 5.06 \text{ cm}$$

(4 marks)

(b) Calculate the area of the circle to 2d.p.

(2 marks)

$$\begin{aligned} \pi r^2 &= A \\ 3.142 \times 5.06 \times 5.06 \\ &= \underline{\underline{80.45 \text{ cm}^2}} \end{aligned}$$

(c) Calculate the area of triangle ABC to 2d.p.

(2 marks)

$$\begin{aligned} \frac{25}{2} \\ A &= \sqrt{12.5(2.5)(3.5)(6.5)} \\ A &= \sqrt{710.9375} \\ A &= 26.66 \text{ cm}^2 \end{aligned}$$

(d) Calculate the area of the shaded part to 2 d.p.

(2 marks)

$$\begin{array}{r} 80.45 \\ - 26.66 \\ \hline \underline{\underline{53.79 \text{ cm}^2}} \end{array}$$

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