SECTION I(50 marks)

ATTEMPT ALL THE QUESTIONS IN THIS SECTION

1. On average, the rate of depression of a water pump is 9% per annum. After three complete years it was Kshs. 150,700. Find its value at the start of the three years period. (3 marks)

$A = P\left(1-\frac{r}{100}\right)^n$

$150{,}700 = P\left(1-\frac{9}{100}\right)^3$ M1

$150{,}700 = P(0.91)^3$

$P = \frac{150{,}700}{(0.091)^3}$ M1

$P = 199981.16$ A1

2. John truncated $^7/_9$ to 3 decimal places. Calculate the percentage error resulting from the truncating. (3marks)

$\frac{7}{9} = 0.7777....$

Truncating to 3 d.p. $0.777 = \frac{777}{1000}$

Absolute error $= 7/9 - \frac{777}{1000} = \frac{7}{9000}$ M1

% Error $= \left(\frac{7}{9000} \div 7/9\right) \times 100\%$ M1

$= 0.1\%$ A1

3. Solve the equation $4\sin^2\Theta + 4\cos\Theta = 5$ for $0° \leq \Theta \leq 360°$ Give your answer in degrees. (3marks)

$4\sin^2\theta + 4\cos\theta - 5 = 0$

$4(1-\cos^2\theta) + 4\cos\theta - 5 = 0$ M1

$4 - 4\cos^2\theta + 4\cos\theta - 5 = 0$

$-4\cos^2\theta + 4\cos\theta - 1 = 0$

$4\cos^2\theta - 4\cos\theta + 1 = 0$

$(4\cos^2\theta - 2\cos\theta) - (2\cos\theta + 1) = 0$

$(2\cos\theta - 1)(2\cos\theta - 1) = 0$ M1

$2\cos\theta - 1 = 0$

$\cos\theta = 1/2$

$\theta = 60°, 300°$

Both A1

4. The first term of an arithmetic sequence is $(2x+1)$ and the common difference is $(x+1)$ if the product of the first and the second terms is zero, find the first three terms of the two possible sequences. (3mks)

$a = 2x+1$

$d = x+1$

$t_1 = 2x+1$

$t_2 = (2x+1)+(x+1)$

$= 3x+2$

M_1 for both terms

$(2x+1)(3x+2) = 0 \quad M_1$

$2x+1 = 0$

$x = -\frac{1}{2}$

or

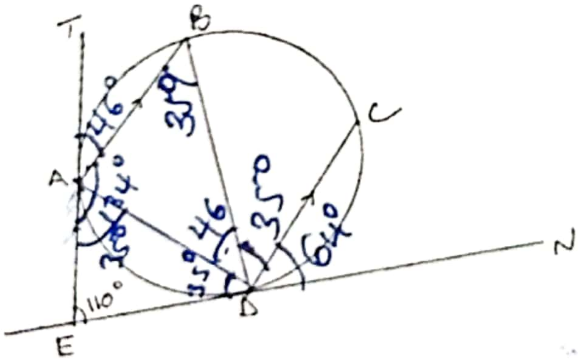
$3x+2 = 0$

$x = -\frac{2}{3}$

A_1 for both

5. TAE and EDN are tangents to a circle at A and D respectively. Line AB and DC are parallel chords, BD is another chord of the circle. Angle TAB is 46°. Find angle CDN giving reasons.

{3 marks}



$\angle ADB = 46^\circ \quad B_1$

$\angle BDC = 35^\circ \quad B_1$

$\angle CDN = 64^\circ \quad B_1$

Award $B_1 B_1 B_1$ if shown on the diagram.

6. Use logarithm table to evaluate. (4marks)

$$\sqrt[4]{\frac{(27\times 0.0293)^2}{(825-94)\div 0.2861}}$$

$$\sqrt{\frac{27\times 0.0293}{731 \div 0.2861}}$$

No	log
27	1.4314
0.0293	$\bar{2}.4669$ +
	$\bar{1}.8983 \times 2$
	$\bar{1}.7966$

No	log
731	2.8639
0.2861	$\bar{1}.4565$ +
	2.3204
	$\bar{1}.7966$ −
	2.3204
	$\cancel{\bar{3}.3204 \div 4}$
	$\bar{3}.4762 \div 4$
2.3394×10^{-1} = 0.23394	$\bar{1}.3691$

7. a) Find the expansion of $\left(1-\frac{x}{3}\right)^7$ in ascending powers of x up to the termin x^2 (1mark)

$$\left(1-\frac{x}{3}\right)^7 = 1.1^7\left(\frac{-x}{3}\right)^0 + 7.1^6\left(\frac{-x}{3}\right)^1 + 21.1^5\left(\frac{-x}{3}\right)^2$$

$$= 1 - \tfrac{7}{3}x + \tfrac{7}{3}x^2 \quad B_1$$

b) use the expansion above to find $(0.99)^7$ to four significant figures (2marks)

$$\left(1-\frac{x}{3}\right)^7 = (1-0.01)^7$$

$$x/3 = 0.01$$

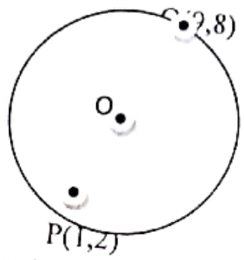
$$x = 0.03$$

$$(0.99)^7 = 1 - \tfrac{7}{3}(0.03) + \tfrac{7}{3}(0.03)^2 \quad M_1$$

$$= 1 - 0.07 + 0.0021$$

$$= 0.9321 \quad A_1$$

8. P and Q are the points on the ends of the diameter of the circle below.



(a) Write down in terms of X and Y the equation of the circle in the form:
$ax^2 + by^2 + x + y + c = 0$ (2 mks)

Diameter $= \sqrt{(9-1)^2+(8-2)^2}$
$= \sqrt{100}$
$= 10$ units
$r = 5$ units
Centre $= \left(\frac{9+1}{2}, \frac{8+2}{2}\right)$
$C(5, 5)$

$(x-5)^2+(y-5)^2=5^2$ M1
$x^2-10x+y^2-10y+25=0$ A1

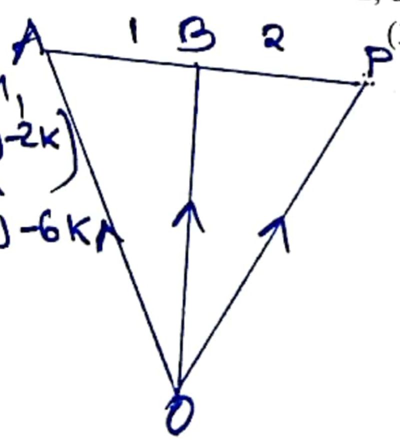
(b) Find the equation of the tangent at Q in the form $ax + by + c = 0$ (2 mks)

Gradient of diameter $= \frac{3}{4} = \left(\frac{8-2}{9-1}\right)$
gradient of tangent $= -\frac{4}{3}$

$\frac{y-8}{x-9} = -\frac{4}{3}$ M1
$4x+3y-60=0$ A1
$4x+3y-60=0$

9. Given that $\overrightarrow{OA} = 3i + 2j - 4k$ and $\overrightarrow{OB} = 4i+5j-2k$ and that p divides AB in the ratio 3: -2, determine the position vector of p in terms of i, j and k (3mks)

$\overrightarrow{OP} = -2a+3b$ M1
$= -2(3i+2j-4k)+3(4i+5j-2k)$ M1
$= -6i-4j+8k+12i+15j-6k$
$= 6i+11j+2k$ A1



10. The masses to the nearest kg of 50 adults were recorded as follows:

Mass (kg)	Frequency (f)	c.f
45 – 50	2	2
51 – 56	10	12
57 – 62	11	23
63 – 68	20	43
69 – 74	6	49
75 – 80	1	50

Calculate the quartile deviation. (3mks)

lower quartile Q_1 $\frac{1}{4} \times 50 = 12.5^{th}$

$56.5 + \left(\frac{12.5 - 12}{11}\right) \times 6$

$= 56.77$ kg. M1 for both Q_1 & Q_3

Upper Quartile Q_3 $= \frac{3}{4} \times 50 = 37^{th}$

$62.5 + \left(\frac{37.5 - 23}{20}\right) \times 6$

$= 66.85$ kg

Quartile deviation

$= \frac{1}{2} \times (66.85 - 56.77)$ M1

$= 5.04$ A1

11. Machine A can complete a piece of work in 6 hours while machine B can complete the same work in 10 hours. If both machines start working together and machine A breaks down after two hours, how long will it take machine B to complete the rest of the work. (3mks)

Both in 1hr $= \frac{1}{10} + \frac{1}{6} = \frac{4}{15}$ M1

Both in 2hrs $= 2 \times \frac{4}{15} = \frac{8}{15}$

Remaining work $= 1 - \frac{8}{15} = \frac{7}{15}$ M1

If $\frac{1}{6} \rightarrow 1$hr

$\frac{7}{15} \rightarrow \frac{7}{15} \times 1 \times 6 = \frac{14}{5} = 2\frac{4}{5}$hrs. A1

2 hrs 48min.

12. Without using tables, rationalize the denominator in

$\frac{2\tan 45^0 - \tan 60^0}{4\tan 45^0 \operatorname{Sin} 30^0 - \sqrt{3}}$

(3 marks)

$$\frac{2(1)-\sqrt{3}}{4(1)(\frac{1}{2})-\sqrt{3}} \quad M_1$$

$$\frac{2-\sqrt{3}}{2-\sqrt{3}} \quad M_1$$

$$=1 \quad A_1$$

or $\frac{2-\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$$\frac{4+2\sqrt{3}-2\sqrt{3}-3}{4+2\sqrt{3}-2\sqrt{3}-3}$$

$$=\frac{1}{1}=1$$

13. Make n the subject of the formula (3 marks)

$$W = \frac{x^2}{(m-n)(m+n)}$$

$$W = \frac{x^2}{m^2-n^2} \quad --- M_1$$

$$w(m^2-n^2) = x^2$$

$$wm^2 - wn^2 = x^2$$

$$wm^2 - x^2 = wn^2$$

$$n^2 = \frac{wm^2-x^2}{w} \quad -- M_1$$

$$n = \pm\sqrt{\frac{wm^2-x^2}{w}} \quad --- A_1$$

A_0 if $\pm$ is missing

or

$$n = \pm\sqrt{\frac{x^2-wm^2}{-w}}$$

14. In a transformation, an object with area $9cm^2$ is mapped onto an image whose area is $54cm^2$. Given that the matrix of transformation is $\begin{bmatrix} x & x-1 \\ 2 & 4 \end{bmatrix}$ find the value of x (3mks)

$$\det = 4(x) - 2(x-1)$$
$$= 4x - 2x + 2$$
$$= 2x + 2$$

$$6 = 2x + 2$$
$$4 = 2x$$
$$2 = x$$

$$\det = \frac{A_{image}}{A_{object}}$$
$$= \frac{54}{9} = 6$$

15. P varies as the cube of Q and inversely as the square root of R. If Q is increased by 20% and R decreased by 36%, find the percentage change in P. (3mks)

$$P = \frac{KQ^3}{\sqrt{R}}$$

Accept alternative method.

$$P_1 = \frac{K(1.2Q)^3}{\sqrt{0.64R}}$$
$$= \frac{1.728KQ^3}{0.8\sqrt{R}}$$

$$P_1 = \frac{2.16KQ^3}{\sqrt{R}}$$

$$\% \text{ change in } P = \left(\frac{P_1 - P}{P}\right) \times 100\%$$
$$= \frac{2.16 - 1}{1} \times 100\%$$
$$= 116\%.$$

16. An arc subtends an angle of 0.9 radians. If radius of circle is 13cm, find the length of the arc. (3 marks)

$$1^c = 57.3^\circ$$

~~$0.9^c \Rightarrow 162^\circ$~~

$$0.9^c = 51.57^\circ . \ M_1$$

Accept equivalent

$$L = \frac{\theta}{360} 2\pi r$$
$$= \frac{51.57}{360} \times 2 \times \frac{22}{7} \times 13 \ M_1$$
$$= 11.71 cm. \ A_1$$

SECTION II (50 MARKS)

(ANSWER ANY FIVE QUESTIONS FROM THIS SECTION)

17. The table below shows the Kenya tax rates in a year

Income (Ksh per annum)	**Tax rate (per £)**
1 – 116,160	10%
116,161 – 225,600	15%
225,601 – 335,040	20%
335,041 – 444,480	25%
Over 444,481	30%

In that year, Ushuru earned a basic salary of Ksh 30000 per month. In addition, he was entitled to a medical allowance of Ksh 2,800 per month and a traveling allowance of Ksh 1800 per month. He is housed by the employer and pays a nominal rent of 2000. He also claimed a monthly family relief of Ksh 1056. Other monthly deductions were union dues Ksh 445, WCPS Ksh 490, NHIF Ksh 320, COOP shares Ksh 1000 and risk fund Ksh 100

Calculate:

(a) Ushuru's annual taxable income. (2marks)

T. Income = Sh. $30000 + \left(\frac{15}{100} \times 30000\right) + 2800 + 1800 - 2000$ M1

= Sh. 37100 A1

(b) The tax paid by Ushuru in that year. (5marks)

Slabs
1st $116160 \times 0.1 = 11616$
2nd $109440 \times 0.15 = 16416$ } B1
3rd $109440 \times 0.20 = 21880$
4th $109440 \times 0.25 = 27360$ } B1
Remaing $720 \times 0.30 = 216$ B1
Gross tax = 77488 B1
Net tax = $77488 - 12672$ = Sh. 64816 B1

(c) Ushuru's net income in that year (3marks)

Total deductions = $5401.33 + 445 + 490 + 320 + 1000 + 100 = 7756.33$ M1

Accept alternative.

net Salary per month $34600 - 7756.33$
= Sh. 26843.67 M1

net salary pa Sh. 322124.04 A1

18. The masses of 50 loaves of bread were taken and recorded as in the table blow.

Mass (gms)	470-479	480-489	490-499	500-509	510-519	520-529	530-539
No. of loaves	1	3	11	21	8	4	2

a. Using an assumed mean of 504.5, calculate the mean mass (3mks).

Mass class	x	f	d = x−504.5	d²	fd	fd²
470-479	474.5	1	−30	900	−30	900
480-489	484.5	3	−20	400	−60	1200
490-499	494.5	11	−10	100	−110	1100
500-509	504.5	21	0	0	0	0
510-519	514.5	8	10	100	80	800
520-529	524.5	4	20	400	80	1600
530-539	534.5	2	30	900	60	1800
		Σf=50			Σfd=20	Σfd²=7400

Σfd → B1
All values correct
fd² → B1
Σfd² → B1

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

$$\bar{x} = 504.5 + \frac{20}{50} \text{ M1}$$

$$= 504.9 \text{ A1}$$

b. Calculate the

i. variance. (4mks)

$$\text{Var } S^2 = \frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2$$

$$= \frac{7400}{50} - \left(\frac{20}{50}\right)^2 \text{ M1}$$

$$S^2 = 148 - 0.16$$

$$= 147.84 \text{ A1}$$

ii. Calculate the standard deviation. (2mks)

$$S = \sqrt{147.84} \text{ M1}$$

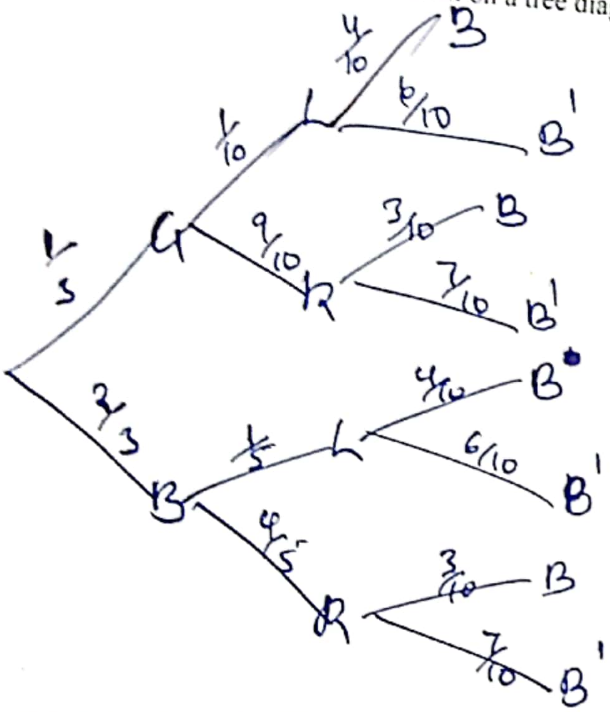
$$= 12.16 \text{ A1}$$

c. If 5 is added to each score and then divided by 3, write down the new standard deviation. (1mk)

$$S = \frac{12.16}{3} = 4.053 \text{ B1}$$

19. In chemistry form 4 classes, $^1/_3$ of the class are girls and the rest boys, $^4/_5$ of the boys and $^9/_{10}$ of the girls are right handed while the rest are left handed. The probability that a right-handed student breaks a conical flask in any practical session is $^3/_{10}$ and the corresponding probability of a left-handed student $^4/_{10}$. The probabilities are independent of the students gender.

(a) Represent the above information on a tree diagram with independent probabilities. (2 marks)



B_2 all value correct

(b) Determine the probability that student chosen at random form the class is left handed and does not break a conical flask in simplest form. (3 marks)

P(GLB') or P(BLB') B_1

$= \left(\frac{1}{3} \times \frac{1}{10} \times \frac{6}{10}\right) + \left(\frac{2}{3} \times \frac{1}{5} \times \frac{6}{10}\right)$ M_1

$\frac{6}{300} + \frac{12}{150} = \frac{30}{300} = \frac{1}{10}$ A_1

(c) Determine the probability that a conical flask is broken in any chemistry practical session in simplest form. (3 marks)

P(GLB) or P(GRB) or P(BLB) or P(BRB) B_1

$\left(\frac{1}{3} \times \frac{1}{10} \times \frac{4}{10}\right) + \left(\frac{1}{3} \times \frac{9}{10} \times \frac{3}{10}\right) + \left(\frac{2}{3} \times \frac{1}{5} \times \frac{4}{10}\right) + \left(\frac{2}{3} \times \frac{4}{5} \times \frac{3}{10}\right)$ $M1$

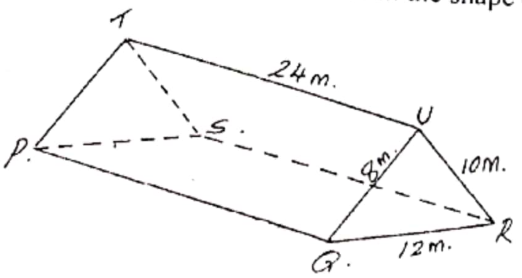
$\frac{4}{300} + \frac{27}{300} + \frac{8}{150} + \frac{24}{150} = \frac{19}{60}$ $A1$

(d) Determine the probability that a conical flask is not broken by a right-handed student in the simplest form. (2 marks)

P(GRB') or P(BRB')

$\left(\frac{1}{3} \times \frac{9}{10} \times \frac{7}{10}\right) + \left(\frac{2}{3} \times \frac{4}{5} \times \frac{7}{10}\right) = \frac{175}{300} = \frac{7}{12}$ M_1 A_1

20. The roof of a ware house is in the shape of a triangular prism as shown below



Calculate

(a) The angle between faces RSTU and PQRS (3mks)

$8^2 = 12^2 + 10^2 - 2(12)(10)\cos R.$

$64 = 144 + 100 - 240\cos R.$

$-180 = -240\cos R$

$0.75 = \cos R$

$41.41^\circ = R$

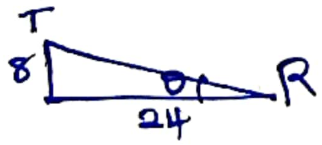
(b) The space occupied by the roof (3mks)

Volume = Cross-section Area x Length.

$= \frac{1}{2} \times 12 \times 10 \sin 41.41 \times 24$

$= 952.48 cm^3.$

(c) The angle between the plane QTR and PQRS (4mks)



$\tan\theta = \frac{8}{24}$

$\theta = 18.43^0$

21. A plane leaves an airport A (41.5^0N, 36.4^0W) at 9:00am and flies due north to airport B on latitude 53.2^0N. Taking π as $\frac{22}{7}$ and the radius of the earth as 6370Km,

a) Calculate the distance covered by the plane in km (4mks)

$53.2 - 41.5 = 11.7^0$ B1

Dist $= \frac{11.7}{360} \times 2 \times \frac{22}{7} \times 6370$ M1

$= 1301.3$ Km. A1

b) The plane stopped for 30minutes to refuel at B and flew due east to C, 2500km from B. Calculate:

i) position of C (3mks)

$\frac{\theta}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 53.2 = 2500$ km.

$=$ ~~2500~~.

$\theta = 37.5^0$.

longitude $= 37.5^0 - 36.4^0 = 1.1^0$.

Z (53.2^0N, 1.1^0E)

ii) The time the plane lands at C if its speed is 500km/h (3mks)

Time $= \frac{1301.3}{500} + \frac{1}{2} + \frac{2500}{500}$

$=$ 8hours 6 minutes.

$37.5^0 \times 4 = 150$min $\rightarrow$ 2hrs 30min.

9.00 + 2h30min + 8hrs 6min.

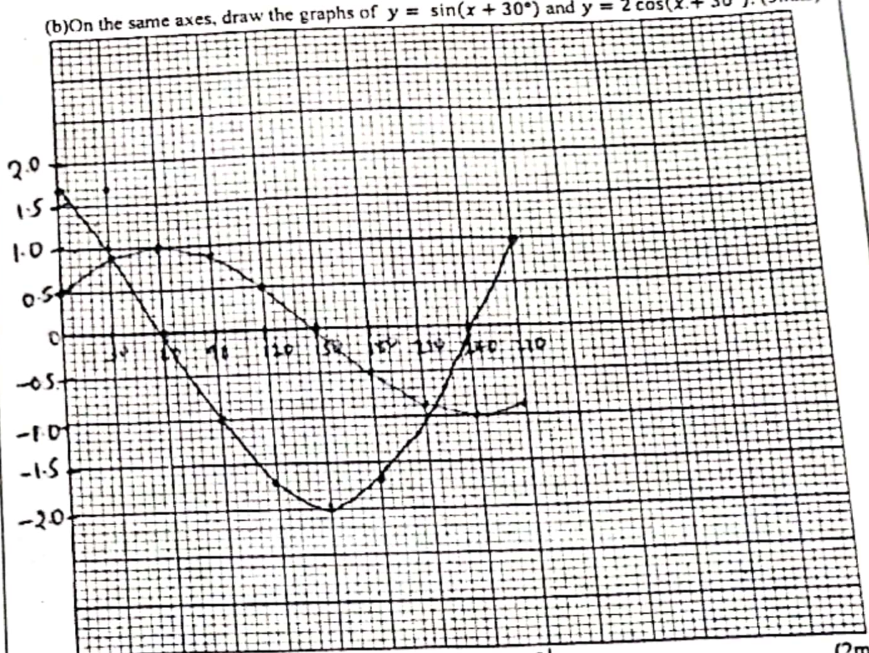
7.36P.m.

22

21. (a) Complete the table below to 2 dp.

$x°$	0	30	60	90	120	150	180	210	240	270
$\sin(x + 30°)$	0.50	0.87	1	0.86	0.5	0	-0.50	-0.87	-1	-0.87
$2\cos(x + 30°)$	1.73	1	0	-1	-1.73	-2	-1.73	-1	0	1

(b)On the same axes, draw the graphs of $y = \sin(x + 30°)$ and $y = 2\cos(x + 30°)$. (5mks)



(c) State the amplitude and period of each wave. (2mks)

	$\sin(x+30)$	$2\cos(x+30)$
Period	360	360
Amplitude	1	2.

(d)Use the graph to solve the equation $2\cos(x + 30°) = \sin(x + 30°)$. (1mk)

216 and 32^0.

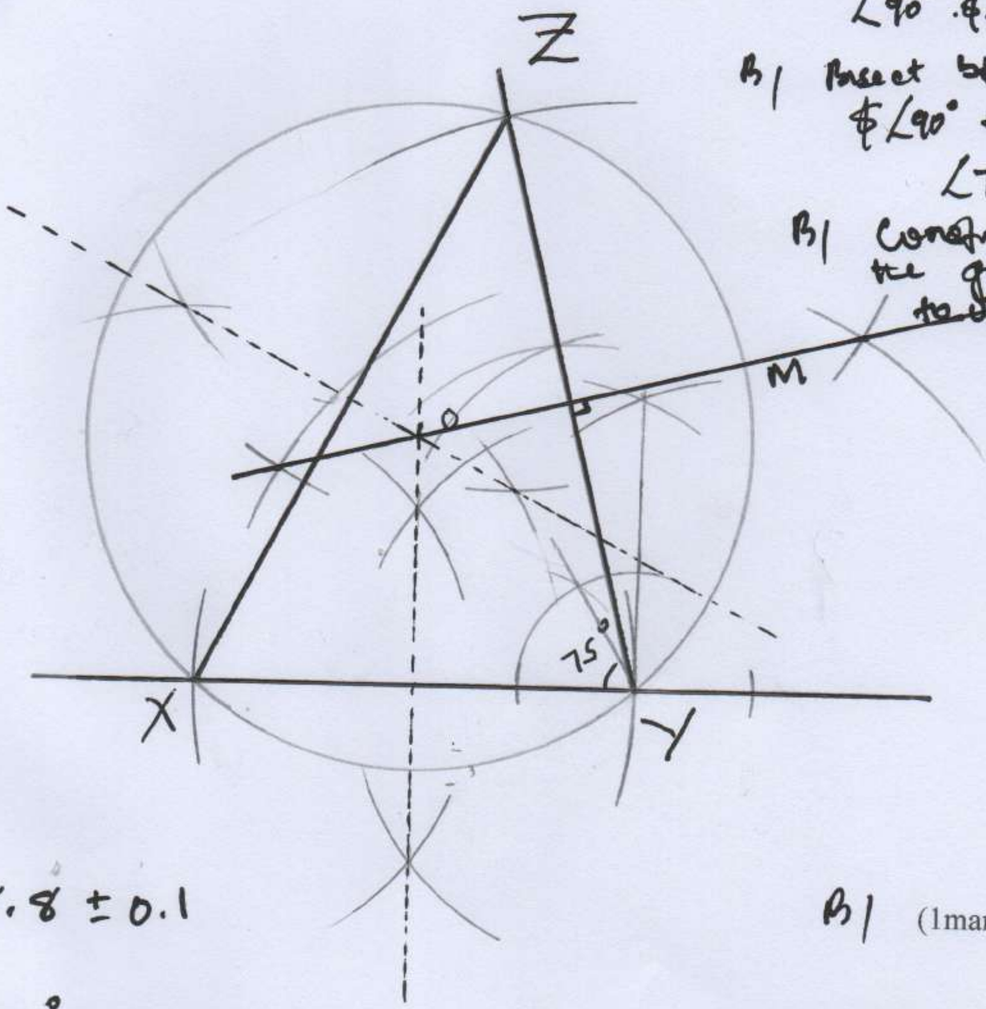
23. a. Using a ruler, a pair of compasses only construct triangle XYZ such that XY = 6cm, YZ = 8cm and $\angle XYZ = 75^\circ$

(3marks)

B1 Construction of $\angle 90^\circ$ & $\angle 60^\circ$

B1 Bisect between $\angle 60^\circ$ & $\angle 90^\circ$ to obtain $\angle 75^\circ$

B1 Construction of the given sides to obtain ΔXYZ



(b) Measure

i. line XZ $= 8.8 \pm 0.1$ B1 (1mark)

ii. $\angle XZY = 42^\circ \pm 1$ B1 (1mark)

(d) Draw a circle that passes through X, Y and Z. (2marks)

B1 - Bisecting any 2-sides

B1 - Circumcircle.

(d) A point M moves such that it is always equidistant from Y and Z. construct the locus of M and define the locus

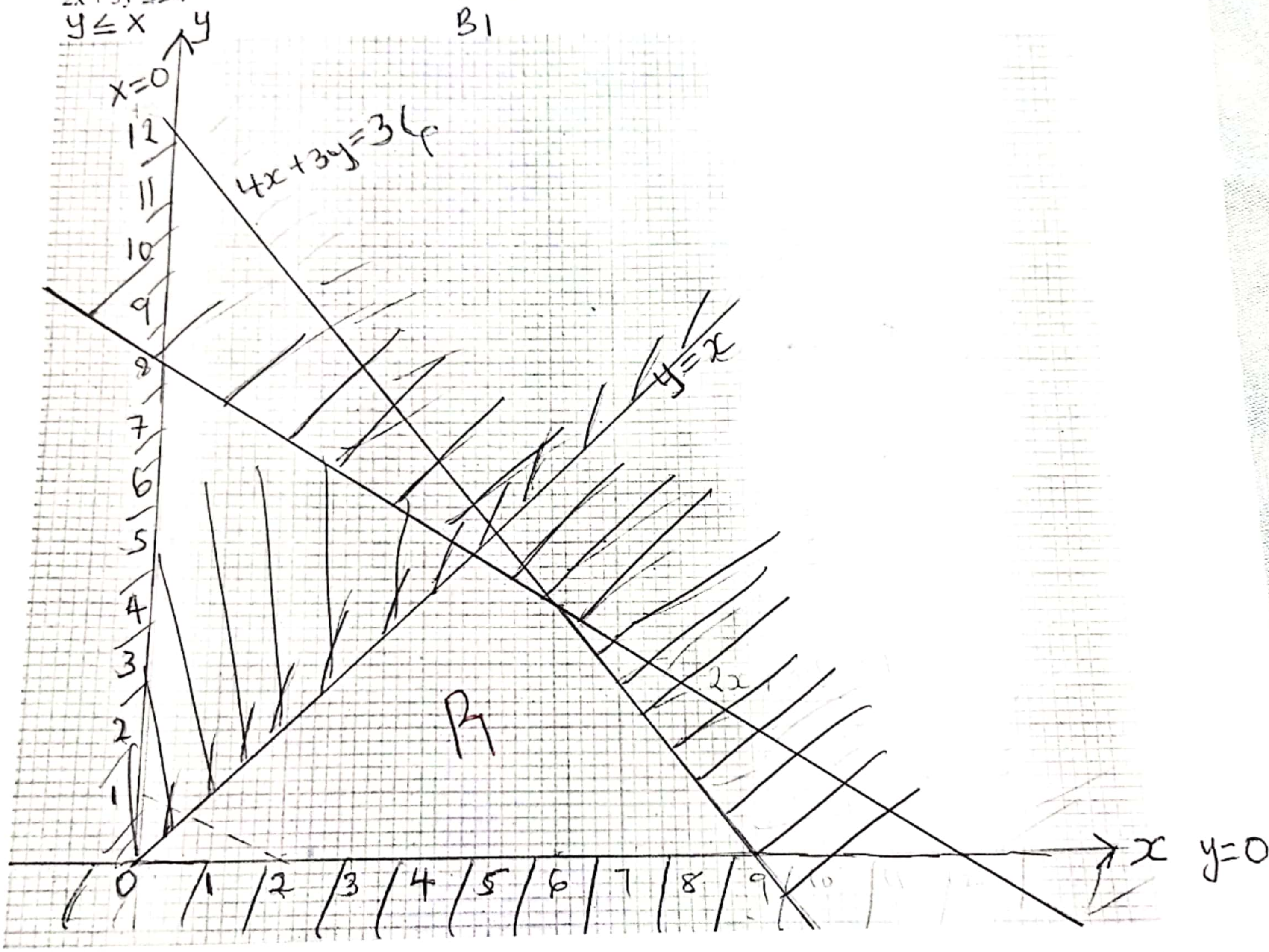
(3marks)

M - Perpendicular bisector of YZ ✓ B1 (Defining)

B1 Bisecting YZ

B1 Indicating the locus M.

24. $4x + 3y < 36$	B1
$2x + 3y \le 24$	B1
$y \le x$	B1



$X \ge 0$, $y \ge 0$	B1	for correct drawing of lines and shading
$Y \le x$	B1	for correct drawing of line and shading
$4x + 3y \le 36$	B1	"
$2X + 3Y \le 24$	B1	"
Profit function $4x + 8y$	B1	
Maximum profit at (6,4)	B1	
Should hire 6 type A and 4 type B machines	B1	

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