Name....MARKING SCHEME..........Adm no.........Class...

Index No...........................… Signature…….:...................….

121/2
Mathematics Paper 2
Form 4
2 ½ Hours
End of Term 1 –September 2021

KASSU JET EXAMINATIONS 2021
Kenya Certificate of Secondary Education (K.C.S.E)

INSTRUCTIONS TO CANDIDATES

➤ Write your name and Admission number in the spaces provided at the top of this page.
➤ This paper consists of two sections: Section I and Section II.
➤ Answer *ALL* questions from section I and *ANY FIVE* from section II
➤ All answers and workings must be written on the question paper in the spaces provided below each question.
➤ Show all the steps in your calculation, giving your answer at each stage in the spaces below each question.
➤ Non – Programmable silent electronic calculators and KNEC mathematical tables may be used, except where stated otherwise.
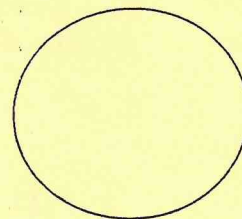
FOR EXAMINERS USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION II

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL

1

SECTION I (Answer **ALL** the questions in this section)

1. Migwambo and Abraham can do a piece of work together in 15 days. Abraham working alone can do the same piece of work in 20 days. How long will Migwambo working alone take to do the same work? (3 marks)

Let Migwambo take y days

$$\frac{1}{y} + \frac{1}{20} = \frac{1}{15} \checkmark$$

$$3y + 60 = 4y \checkmark$$

$$4y - 3y = 60$$

$$y = 60 \checkmark$$

$$\therefore \text{Migwambo takes } 60 \text{ days}$$

2. Given the measurements P=12.5cm, Q=5.0cm and R=2.5cm, find in 4 significant figures, the percentage error in $\frac{P}{QR}$ (3 marks)

Max. P. value $= \dfrac{12.55}{4.95 \times 2.45} = 1.034838178 \checkmark$

Min P. value $= \dfrac{12.45}{5.05 \times 2.55} = 0.966802562 \checkmark$

Actual value $= \dfrac{12.5}{5.0 \times 2.5} = 1$

Error $= 0.034017808 \times 100\% = 3.402\% \checkmark$

3. Make b the subject of the formula in $S = \frac{\sqrt{3t(b-x)}}{9}$ (3 marks)

$$81S^2 = 3t(b-x) \checkmark$$

$$3bt = 81S^2 + 3tx \checkmark$$

$$b = \frac{81S^2 + 3tx}{3t}$$

$$b = \frac{27S^2 + tx}{t} \quad \text{or} \quad b = \frac{27S^2}{t} + x \checkmark$$

4. Solve for x given $M = \begin{pmatrix} -2 & -3x \\ x & 6 \end{pmatrix}$ is a singular matrix hence state the possible matrices (3 marks)

$6(-2) - (-3x^2) = 0$

$3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

$(x-2)(x+2) = 0$

$x = 2 \text{ or } -2$

$\begin{pmatrix} -2 & -6 \\ 2 & 6 \end{pmatrix}$

$\begin{pmatrix} -2 & 6 \\ -2 & 6 \end{pmatrix}$

5. Find the exact value for x in the following equation $\log_2(x+4) = 2 - \log_2 \frac{1}{8}$ (3 marks)

$\log_2(x+4) = \log_2 4 - \log_2 \frac{1}{8}$

$\log_2(x+4) = \log_2 \left(\frac{4 \times 8}{1} \right)$

$x + 4 = 32$

$x = 28$

6. Solve by completing the square: (3 marks)
$2\frac{1}{2}x^2 + 1\frac{2}{3}x - 1\frac{1}{4} = 0.$

$30x^2 + 20x - 15 = 0$

$x^2 + \frac{2}{3}x + c = \frac{1}{2} + c$

$\left(x + \frac{1}{3} \right)^2 = \frac{11}{18}$

$x + \frac{1}{3} = \pm 0.78173596$

$x = -\frac{1}{3} \pm 0.78173596$

$x = 0.4484 \quad \text{or} \quad -1.115$

7. The cash price of a Samsung S20 is Ksh. 125,000. Yianti bought the phone on hire purchase terms by paying a deposit of Ksh. 70,000 and the balance by 24 equal monthly instalments of Ksh. 3,000. Find the rate of carrying charge per year giving your answer to 4 significant figures. (3marks)

New principal $= 125000 - 70,000$

$\qquad = 55,000$

Amount $= 24 \times 3000 = \text{sh} 72000$

$$72\,000 = 55\,000 \left(1 + \frac{r}{100}\right)^2 \checkmark$$

$$1 + \frac{r}{100} = 1.144155107 \checkmark$$

$$r = 14.42\% \text{ p.a} \checkmark$$

8. An arc length of 18.5 cm subtends an of angle of 1.2^c at the centre of the circle .Find the diameter of the circle to one decimal place. (4 marks)

$$1.2^c = \frac{216}{\pi}$$

$$\frac{216}{360\pi} \times 2\pi r = 18.5 \checkmark \qquad D = 30.8 \checkmark$$

$$r = \frac{18.5 \times 360}{216} \times \frac{1}{2} \checkmark$$

$$r = 15\frac{5}{12} \checkmark$$

Diameter $= 15\frac{5}{12} \times 2 \checkmark$

9. Solve for α in the equation $3\cos 5\alpha^0 = -\frac{3\sqrt{2}}{2}$ where $0^0 \leq \alpha \leq 100^0$ (3 marks)

$$5\alpha = 45^\circ \checkmark$$

$$5\alpha = 135^\circ, 225^\circ, 495^\circ, \cancel{585^\circ}, \cancel{855}, \cancel{945}$$

$$\alpha = 27^\circ, 45^\circ, 99^\circ \checkmark$$

4

10. John spent one-third of his money on bread and two-fifths of the remainder on soda. He used the balance to purchase four pens at a total cost of sh. 300. How much money did he have at the beginning? (2 marks)

$$Soda = \frac{4}{15}$$

$$Bread \ and \ soda = \frac{3}{5}$$

$$Balance = \frac{2}{5} \quad \checkmark M_1$$

$$\frac{2}{5} = 300$$

$$\frac{5}{5} = ?$$

$$\frac{5}{5} \times 300 \times \frac{5}{2}$$

$$= ksh \ 750 \quad \checkmark A_1$$

11. In the figure below OS is the radius of a circle centre O. Chords SQ and TU are extended to meet at P and OR is perpendicular to QS at R. OS = 61 cm, PU = 50 cm, UT = 40 cm. and PQ = 30
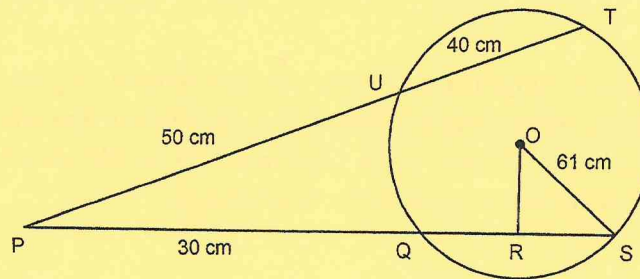
Calculate the length of

a) QS (2 marks)

$$Let \ QS = x$$

$$30(30+x) = 90 \times 40 \quad \checkmark M_1$$

$$x = 90$$

$$\therefore QR = 90 \quad \checkmark A_1$$

b) OR to 2 decimal places (1 mark)

$$OR = \sqrt{61^2 - 45^2} \quad \checkmark M_1$$

$$= \sqrt{1696}$$

$$= 41.18 \ cm \quad \checkmark A_1$$

5

12. Expand $(1 + \frac{1}{2}x)^{10}$ up to the term in x^3 in ascending powers of x. Hence find the value of $(1.005)^{10}$ correct to four decimal places.   (4 marks)

$$1^{10}(0.5x)^0 + 10 \cdot 1^9 (0.5x)^1 + 45 \cdot 1^8 (0.5x)^2 + 120 \cdot 1^7 (0.5x)^3$$

$$1 + 5x + 11.25x^2 + 15x^3$$

$$x = 0.1$$

$$1 + 5(0.1) + 11.25(0.1)^2 + 15(0.1)^3$$

$$1.6275$$

13. The charge, C shillings per person for a certain seminar is partly fixed and partly varies inversely as the total number of N people. Given that when 100 people attend the charge is ksh 8,700 per person while when 35 people attend the charge per person increases by ksh 1300. Calculate the fixed charge.   (3 marks)

$$8700 = k + \frac{a}{100}$$

$$10000 = k + \frac{a}{35}$$

M1 (Formation of equations)

$$100k + a = 870\,000$$
$$- \quad 35k + a = 350\,000$$
$$\overline{\quad 65k = 520,000 \quad}$$

M1 (attempt to solve)

$$k = 8000$$

$$800,000 + a = 870,000$$

$$a = 70,000$$

6

14. Without using a calculator or mathematical table evaluate $\dfrac{2\tan 60^0}{\sin 45^0 - \cos 30^0}$ leaving your answer in simplified form. (3 marks)

$$\frac{2\sqrt{3}}{\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}}$$ ✓ M

$$\frac{8\sqrt{6} + 24}{-2}$$

~~2√3~~

$$-4\sqrt{6} - 12 \quad ✓ A$$

$$\frac{4\sqrt{6}}{2 - \sqrt{6}} \times \frac{2 + \sqrt{6}}{2 + \sqrt{6}} ✓ \quad M$$

15. Use logarithms to evaluate; (3 marks)

$$\sqrt[4]{\frac{\log 6 \times 27.5}{0.46 \times 36}}.$$

No.	Std form	log
0.7782	7.782×10^{-1}	$\bar{1}.8911$
27.5	2.75×10^1	1.4393

✓ M (for all correct logs).

1.3304

| 0.46 | 4.6×10^1 | $\bar{1}.6628$ |
| 36 | 3.6×10^1 | 1.5663 |

$\bar{1}.2191$

1.066 ✓ A 1.066×10^0 $0.1113 \times \frac{1}{4} = 0.02783$ ✓ M

16. The equation of a circle is $x^2 + y^2 + 6x - 10y - 2 = 0$. Determine the co-ordinates of the centre and the area of the circle in terms of π (3 marks)

$$x^2 + 6x + C_1 + y^2 - 10y + C_2 = 2 + 4 + C_2$$

$$(x+3)^2 + (y-5)^2 = 36 \quad ✓ M$$

$$\text{Centre } (-3, 5) \Big\} ✓ M$$

$$\text{Radius} = 6$$

$$\text{Area} = 36\pi \quad ✓ A$$

17. (a). Complete the table below for the function $y = (x + 3)(x + 1)(x - 2)$ (2 marks)

x	−4	−3	−2	−1	0	1	2	3
y	−18	0	4	0	−6	−8	0	18

B_1
B_1

(b). Using the values obtained in the table above draw the graph of $y = (x + 3)(x + 1)(x - 2)$.

(3 marks)

P_1
S_1
C_1
L_1
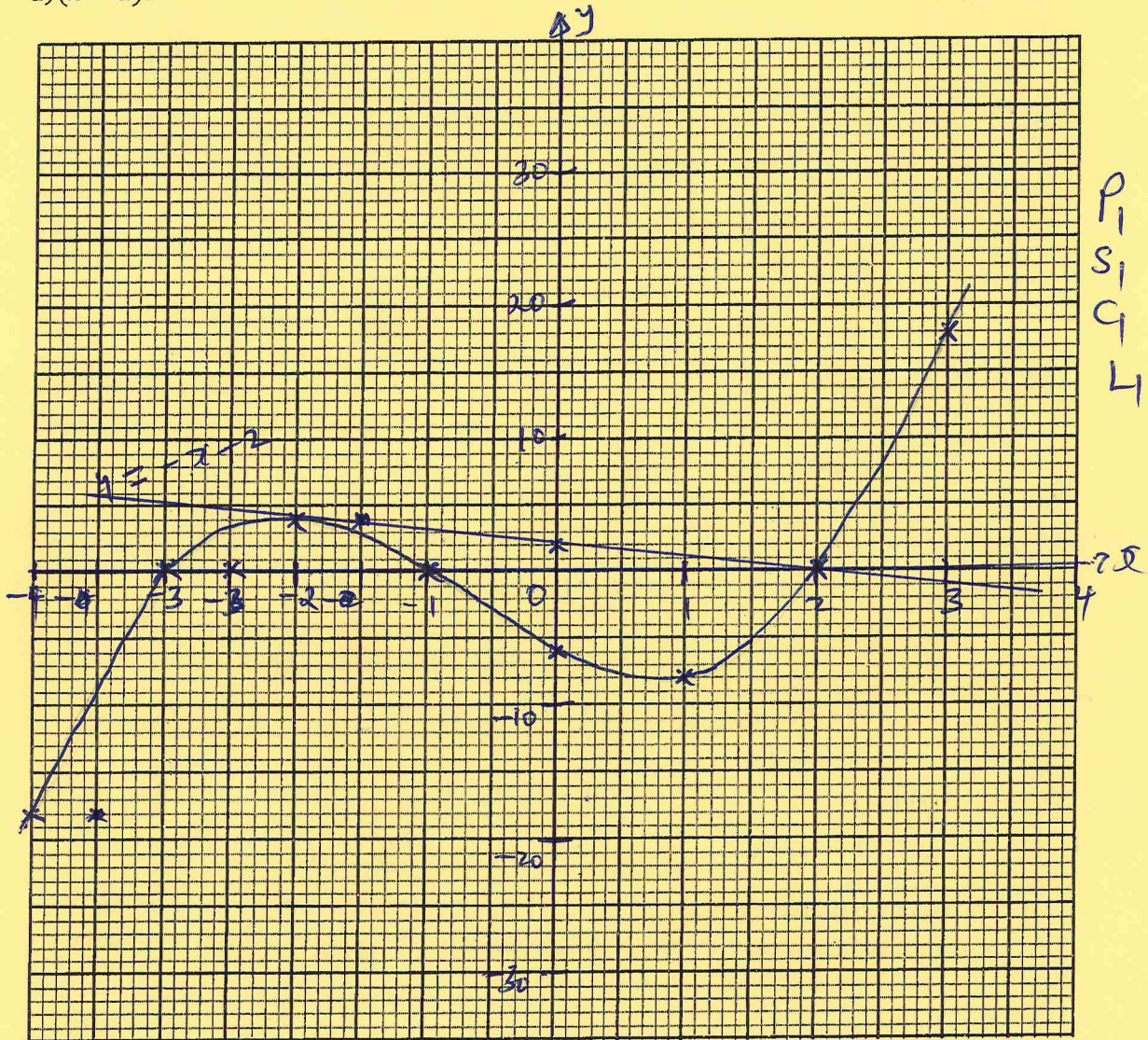
(a) Using your in (b) above, solve the following equations.

   i. $x^3 + 2x^2 - 5x - 6 = 0$ (2 marks)

   ii. $y = 0$ B_1 $x = -3, -1$ or 2 B_1

   iii. $x(x^2 + 2x - 4) = 8$ (3 marks)

$y = -x + 2$ B_1 B_1 for line

$x = -2$ or 2 B_1

8

18. a) Given the series $2 + 4 + 8 + 16 + \text{---------} + 16384$,

(i) Identify the type of series (1 mark)

Geometric series (G.P)

(ii) Find the number of terms in the series (3 marks)

$16384 = 2(2)^{n-1}$

$8192 = 2^{n-1}$

$\log 8192 = (n-1)\log 2$

$n - 1 = \dfrac{\log 8192}{\log 2}$

$n - 1 = 13$

$n = 13 + 1$

$n = 14$

(iii) Calculate the sum terms in the series (3 marks)

$\dfrac{2(2^{14} - 1)}{2 - 1}$

$2 \times \dfrac{(16384 - 1)}{1}$

$= 32766$

b) The first three consecutive terms of an increasing geometric progression are 3, x, and $5\frac{1}{3}$. Find the value of x. (3 marks)
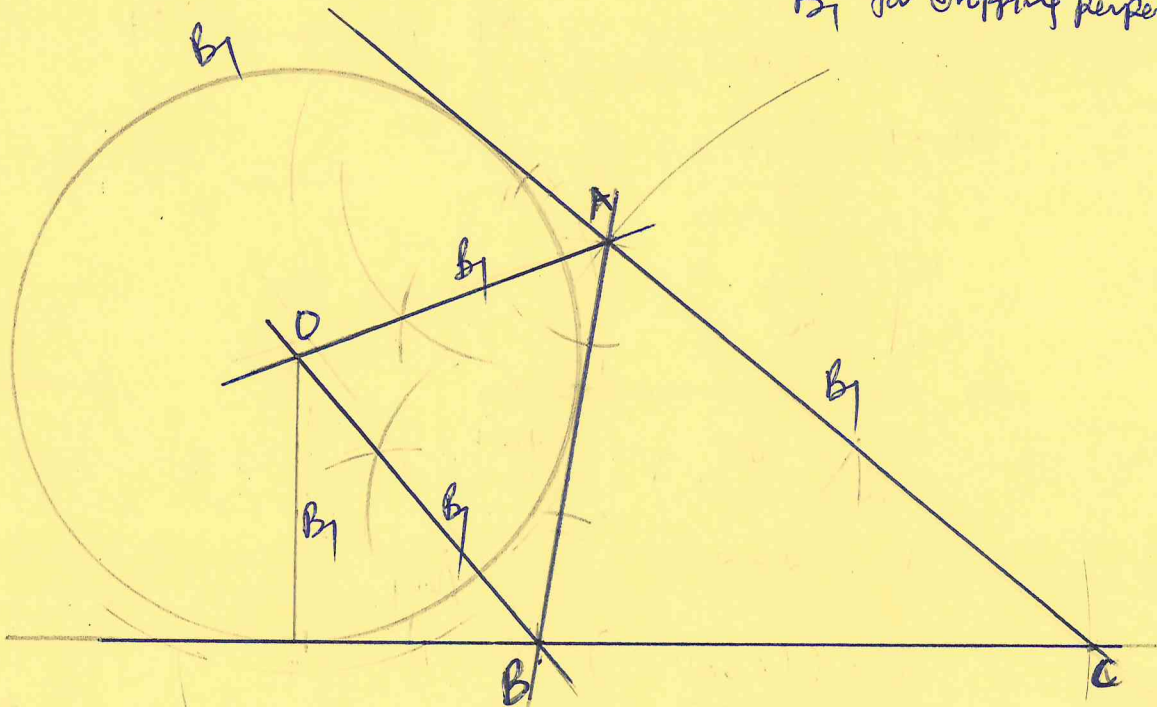
$\dfrac{x}{3} = \dfrac{16}{3x}$

$x^2 - 16 = 0$

$x = 4 \text{ or } -4$

$\therefore x = 4$

9

19. Construct triangle ABC in which $AB = 5.4\ cm$, $BC = 7.4\ cm$ and $AC = 8.4\ cm$.

Construct an escribed circle opposite angle ACB   (5 marks)

B₁ for △ ABC
B₁B₁ for bisectors
B₁ for Circle
B₁ for dropping perpendicular

(a) Measure the radius of the circle   (1 mark)

$3.7 \pm 1°$ B₁

(b) Measure the acute angle subtended at the centre of the circle by AB   (1 mark)

$70°$ B₁

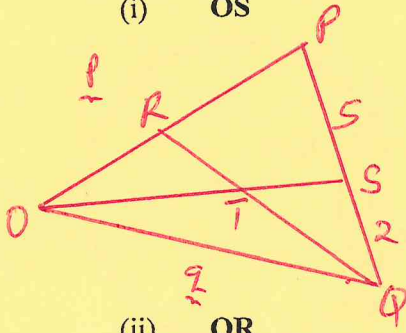(c). Calculate the area of triangle OBA with O as the centre of the circle   (3 marks)

$\frac{1}{2} \times 4.4 \times 4.9 \sin 70°$ = ✓ M₁   B₁ for 4.4 cm and 4.9 cm.

$= 10.13\ cm^2$ ✓ A₁

10

20. A triangle OPQ is such that **OP=p** and **OQ = q**. A point R divides OP in the ratio 1:2 and a point S divides PQ in the ratio 5:2. OS and RQ meet at T. Determine

(a) Express in terms p and q

(i)   **OS**   (1 mark)

$$\vec{OS} = p + \frac{5}{7}(-p+q)$$

$$\frac{2}{7}\underline{p} + \frac{5}{7}\underline{q} \checkmark$$

(ii)   **QR**   (1 mark)

$$QR = \vec{QO} + \frac{1}{3}\vec{OP}$$

$$= -\underline{q} + \frac{1}{3}\underline{p} \checkmark$$

(b) Given that **OT=hOS** and   **RT= kRQ**,   express **OT** in terms of

(i) h, p and q (1 mark)

$$OT = \frac{2}{7}h\underline{p} + \frac{5}{7}h\underline{q} \checkmark$$

(ii) k, p and q (1 mark)

$$OT = OR + RT$$

$$\left(\frac{1}{3} - \frac{1}{3}k\right)\underline{p} + k\underline{q} \checkmark$$

(c) Find the values of h and k (4 marks)

$$\frac{2}{7}h\underline{p} + \frac{5}{7}h\underline{q} = \left(\frac{1}{3} - \frac{1}{3}k\right)\underline{p} + k\underline{q} \checkmark$$

$$6h = 7 - 7k \checkmark B_1$$

$$\frac{5}{7}h = k \checkmark B_1$$

$$h = \frac{7}{11}$$

$$k = \frac{5}{7} \times \frac{7}{11} = \frac{5}{11} \checkmark$$

(d) Find the ratio in which R divides line QT   (2 marks)

$$11RT = 5RQ \checkmark$$

$$11 : -5 \checkmark 11$$

21. The figure below shows two intersecting circles with centres A and B and radii 12cm and 10cm respectively. CD = 8cm and is a common chord. Calculate to one decimal place
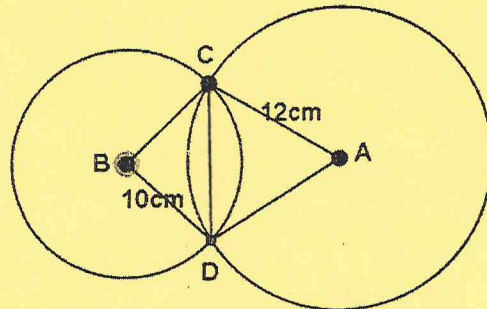
  (a) the area of quadrilateral ACBD (3 marks)

$MB = \sqrt{84}$

$= 9.165$

$AM = \sqrt{128}$

$= 11.313$

$AB = 9.165 + 11.313$

$= 20.48$

$A = \frac{1}{2} \times 8 \times 20.48$

$= 81.9 \, cm^2$

(b) the area of the common region between the intersecting circles. Use $\pi = \frac{22}{7}$

(5 marks)

$\sin \theta = 0.4$

$\theta = 23.58°$

$\angle CBD = 47.16°$

$\sin B = \frac{4}{12}$

$B = 19.47$

$\angle CAD = 38.94°$

$\frac{47.16}{360} \times \frac{22}{7} \times 10 \times 10 - \frac{1}{2} \times 10 \times 10 \sin 47.16$

$41.17 - 36.66 = 4.507$

$\frac{38.94}{360} \times \frac{22}{7} \times 12 \times 12 - \frac{1}{2} \times 12 \times 12 \sin 38.94$

$48.95 - 45.25 = 3.698$

Area of Common region

$= 4.507 + 3.698$

$= 8.2 \, cm^2$

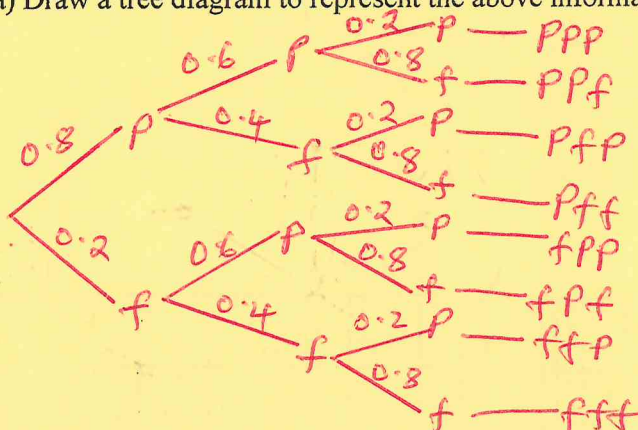(c) the area in quadrilateral ACBD that is not shared by the intersecting circles. (2 marks)

$81.9 - 8.2$

$= 73.7 \, cm^2$

12

22. The probability of Patrick passing his exam is 0.8 that of James is 0.6 while that of Lewis is 0.2.

(a) Draw a tree diagram to represent the above information   (2 marks)

(b) Use your tree diagram to find the probability that;

   i)   Exactly two students pass in the exam   (2 marks)

$$P(PPf) \text{ OR } P(PfP) \text{ OR } P(fPP)$$

$$0.8 \times 0.6 \times 0.8 + 0.8 \times 0.4 \times 0.2 + 0.2 \times 0.6 \times 0.2$$

$$0.384 + 0.064 + 0.024$$

$$0.472$$

   ii) At most two students pass the exam   (2 marks)

$$P(PPf) \text{ OR } P(PfP) \text{ OR } P(Pff) \text{ OR } P(fPP) \text{ OR } P(fPf) \text{ OR } P(ffP)$$

$$0.8 \times 0.6 \times 0.8 + 0.8 \times 0.4 \times 0.2 + 0.8 \times 0.4 \times 0.8 + 0.2 \times 0.6 \times 0.2 + 0.2 \times 0.4 \times 0.2$$

$$0.384 + 0.064 + 0.256 + 0.024 + 0.096 \times 0.016$$

$$0.84$$

   iii) Only one student passes the exam   (2 marks)

$$P(Pff) \text{ OR } P(fPf) \text{ OR } P(ffP)$$

$$0.256 + 0.096 + 0.016$$

$$0.368$$

   iv) At least one passes in the exam   (2 marks)
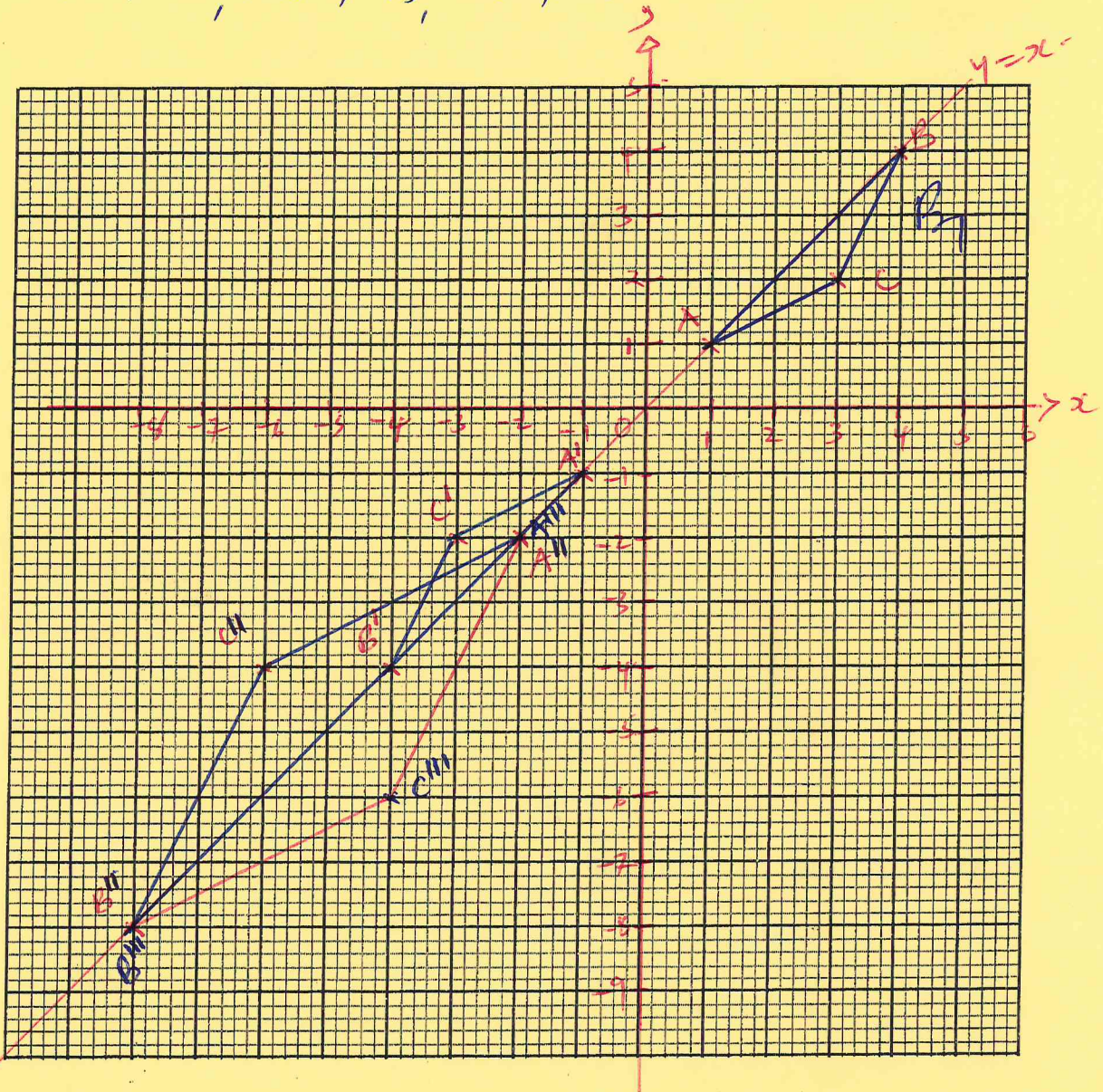
$$1 - P(fff)$$

$$1 - (0.2 \times 0.4 \times 0.8)$$

$$1 - 0.064 = 0.936$$

13

23. (a) Triangle ABC with vertices $A(1,1)$, $B(4,4)$ and $C(3,2)$ is mapped onto triangle $A^I B^I C^I$ by transformation represented $P = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. State the coordinates of $A^I B^I C^I$ hence plot the triangle and its image on the grid provided below.   (3 marks)

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 4 & 3 \\ 1 & 4 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -4 & -3 \\ -1 & -4 & -2 \end{pmatrix} \checkmark B1$$

$A'(-1,-1), \; B'(-4,-4), \; C'(-3,-2) \checkmark B1$

14

(b). $A^{II}B^{II}C^{II}$ is the image of $A^{I}B^{I}C^{I}$ under transformation represented by T= $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. State the coordinates of $A^{II}B^{II}C^{II}$ hence plot the triangle on the same grid. (2 marks)

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{matrix} A^I & B^I & C^I \\ \begin{pmatrix} -1 & -4 & -3 \\ -1 & -4 & -2 \end{pmatrix} \end{matrix} = \begin{matrix} A^{II} & B^{II} & C^{II} \\ \begin{pmatrix} -2 & -8 & -6 \\ -2 & -8 & -4 \end{pmatrix} \end{matrix}$$

$A^{II}(-2,-2),\quad B^{II}(-8,-8),\quad C^{II}(-6,-4)\checkmark$

(for plotting)

(c) $A^{II}B^{II}C^{II}$ is mapped onto $A^{III}B^{III}C^{III}$ by a reflection in the line $y = x$. Plot $A^{III}B^{III}C^{III}$ and hence state the coordinates (for plotting) (2 marks)

$A^{III}(-2,-2),\quad B^{III}(-8,-8)\quad C^{III}(-4,-6)\checkmark$

(d) Find the matrix that represents the transformation that maps triangle $A^{III}B^{III}C^{III}$ onto ABC. (3 marks)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} -2 & -4 \\ -2 & -6 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix} \checkmark$$

$$-2a - 2b = 1 \qquad\qquad -2c - 2d = 1$$
$$-4a - 6b = 3 \qquad\qquad -4c - 6d = 2 \quad \text{(for solving the equation.)}$$

$$b = -\tfrac{1}{2} \qquad\qquad\qquad d = 0$$
$$a = 0 \qquad\qquad\qquad\quad c = -\tfrac{1}{2}$$

$$\text{Matrix} = \begin{pmatrix} 0 & -\tfrac{1}{2} \\ -\tfrac{1}{2} & 0 \end{pmatrix} \checkmark$$

24. The table below shows marks scored by students in a given test

Marks	frequency	cf
70 – 74	4	4
75 – 79	8	12
80 – 84	11	23
85 – 89	15	38
90 – 94	9	47
95 – 99	3	50

(a). Using an assumed mean of 87, Estimate;

i)   The mean   (3 marks)

$$M_1 \text{ (for fd column)}$$

$$87 + \frac{-120}{50} \quad \checkmark M$$

$$87 - 2.4$$

$$84.6 \quad \checkmark A_1$$

ii)   The standard deviation   (4 marks)

x	d = x – 87	fd	fd²
72	–15	–60	900
77	–10	–80	800
82	–5	–55	275
87	0	0	0
92	5	45	225
97	10	30	300
		–120	2500

$$\sqrt{\frac{2500}{50} - \left(\frac{-120}{50}\right)^2} \quad \checkmark M$$

$$\sqrt{50 - 5.76}$$

$$\sqrt{44.24}$$

$$\underline{6.651} \quad \checkmark A_1$$

(b). Calculate the 5th decile   (3 marks)

$$84.5 + \frac{25 - 23}{15} \times 5 \quad \checkmark M$$

$$84.5 + 0.6667 \quad \checkmark M$$

$$85.17 \quad \checkmark A_1$$