Name MARKING SCHEME

Adm. No ______________ Stream ______________

School ______________

BUNAMFAN EXAMINATIONS

Kenya Certificate of Secondary Education

MATHEMATICS
PAPER 121/2
ALT. A
FORM 4
2½ Hrs

Instructions to Candidates

1. Write your name, Admission Number and Stream in the spaces provided at the top of this page.
2. Show all your workings in the spaces provided below each question.
3. This paper contains two sections, Section I and Section II.
4. Answer all the questions in section I and any five questions in section II.
5. All the questions in section II carry equal marks.
6. Negligence and slovenly work will be penalized.
7. Mathematical tables and non-programmable electronic calculators maybe used.

FOR OFFICIAL USE ONLY

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

17	18	19	20	21	22	23	24	Total

GRAND TOTAL

SECTION I (50 Mks)

(Answer all questions in this section)

1. Use logarithms tables to **evaluate**. $\left(\dfrac{130.9}{27.68 \times 100.9}\right)^{2/3}$ (4mks)

No	Log
130.9	~~2.1169~~ 2.1169
27.68	1.4422
100.9	2.0039 +
	3.4461 −
	2.6708

$\bar{2}.6708 \times \dfrac{2}{3}$

$\dfrac{\bar{3}}{3} . \dfrac{3416}{3}$

$\bar{1}.1139$

antilog $= 0.1300$ → Accuracy → A1

All 3 logs – M1

Attempt to add & subtract – M1

multiplying & dividing – M1

2. A trader mixes grade A coffee costing sh 600 per kg, with grade B coffee costing sh. 280 per kg in the ratio 3 : 5. Find the price at which he must sell 1 kg of the mixture to make a profit of 20 %. (4 mks)

Cost

$\left[\frac{3}{8} \times 600\right] + \left[\frac{5}{8} \times 280\right]$ ✓ M1

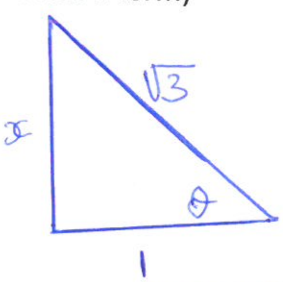
$225 + 175$

$= 400$ ✓ A1

$\frac{120}{100} \times 400$ ✓ M1

$=$ Sh. 480 ✓ A1

3. Given that $\cos\theta = \frac{1}{\sqrt{3}}$, find the value of $\dfrac{\tan\theta + \sin\theta}{\cos\theta}$ in its simplest form. (Leave your answer in surd form) (3mks)



$x = \sqrt{(\sqrt{3})^2 - 1^2}$

$= \sqrt{3 - 1}$

$= \sqrt{2}$ ✓ B1

$\dfrac{\sqrt{2} + \frac{\sqrt{2}}{\sqrt{3}}}{\frac{1}{\sqrt{3}}}$ ✓ M1

$\dfrac{\sqrt{6} + \sqrt{2}}{\sqrt{3}} \times \dfrac{\sqrt{3}}{1}$

$= \sqrt{6} + \sqrt{2}$

4. Determine the equation of the normal to the curve $y = 3x^2 - 4x + 5$ at the point (1, 4). (3mks)

$y = 3x^2 - 4x + 5$

$\frac{dy}{dx} = 6x - 4$ ✓ M1

(1, 4)

$\frac{dy}{dx} = (6 \times 1) - 4$

$= 2$

$2 \times g_H = -1$

$g_H = -\frac{1}{2}$

$\frac{y-4}{x-1} = -\frac{1}{2}$ ✓ M1

$y = -\frac{1}{2}x + \frac{1}{2} + 4$

$y = -\frac{1}{2}x + 4\frac{1}{2}$ ✓ M1

5. Water flows from a pipe at the rate of 250 litres per minute. If the pipe is used to drain a tank full of water measuring 3.2m by 2.5m by 2m, how long would it take to drain the tank completely when it is $\frac{3}{4}$ full? (3 mks)

Volume $= [3.2 \times 2.5 \times 2] \times \frac{3}{4}$ ✓ M1

$= 16 \times \frac{3}{4} = 12m^3$.

Volume $= 12 \times 1000$

12000L

Time $= \frac{12000}{250}$ ✓ M1

$= 48$ minutes

6. Make N the subject of the formula $t = \frac{5P-N}{3N-P}$ (2mks)

$t \times (3N-P) = \frac{5P-N}{3N-P} \times (3N-P)$ ✓ M1

multipling both sides by (3N-P).

$3Nt - tP = 5P - N$

$3Nt + N = 5P + tP$

$N(3t+1) = 5P + tP$

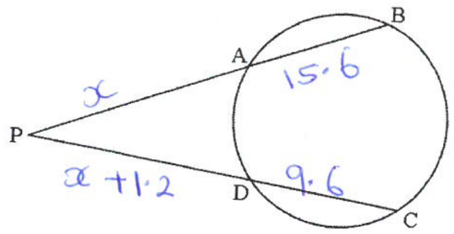
$N = \frac{5P + tP}{3t+1}$ ✓ A1

7. Determine the period and amplitude of the function. $y = 4 \sin (2x - 20°)$ (2 mks)

Amplitude $= 4$ ✓ B1

Period $= \frac{360}{2} = 180°$ ✓ B1

8. In the figure below, **PA** is 1.2cm shorter than **PD**. Given that **AB** = 15.6cm, **CD** = 9.6cm,



Determine the length of **PA**.

(3mks)

$[15.6 + x]x = [x + 1.2 + 9.6][x + 1.2]$ ✓M1

$15.6x + x^2 = [x + 10.8][x + 1.2]$

$15.6x + x^2 = x^2 + 12x + 12.96$

$x = \frac{12.96}{3.6}$ ✓M1 $= 3.6$ cm ✓A1

9. Without using logarithms table or calculator, solve for x in;

log 5 – 2 + log(2x + 10) = log (x - 4)

(3mks)

$\log 5 + \log(2x+10) - \log(x-4) = 2.$

$\log \frac{5(2x+10)}{x-4} = 2$ ✓M1

$10^2 = \frac{5(2x+10)}{x-4}$ ✓M1

$x = 5$ ✓A1

10. In an arithmetic progression, the 20th term is 92 and the sum of the first 20 terms is 890. Calculate;

(a) The first term

(2 mks)

$a + 19d = 92$

$\frac{20}{2}\{2a + 19d\} = 890$ M1

$a + 19d = 92$

$2a + 19d = 89$

$-a = 3$

$a = -3$ ✓A1

(b) The common difference

(1 mk)

$a + 19d = 92$

$d = \frac{92 + 3}{19}$

$d = 5$ ✓B1

11. Solve for θ in the equation $Sin(3\theta + 120^0) = \frac{\sqrt{3}}{2}$ for $0 \leq \theta \leq 180^0$. (3mks)

$3\theta + 120 = \sin^{-1}\frac{\sqrt{3}}{2} = 60^0$ ✓ B1 for 60^0

$3\theta + 120 = 60^0, 120^0, 420^0, 480^0, 780^0$ ✓ M1 (for First four.)

$\theta = -20^0, 0^0, 100^0, 120^0, 220^0$.

$\therefore \theta = 0^0, 100^0, 120^0$ ✓ (A1 for all 3)

12.(a) **Expand** and **simplify** the expression $\left(4x - \frac{y}{2}\right)^5$ up to the third term. (2mks)

$1(4x)^5 \cdot \left(\frac{-y}{2}\right)^0 + 5(4x)^4\left(-\frac{y}{2}\right)^1 + 10(4x)^3\left(-\frac{y}{2}\right)^2$ ✓ M1

$(1 \times 1024x^5 \times 1) + \left(5 \times 256 \times x^4 \times -\frac{y}{2}\right) + [10 \times 64 \times x^3 \cdot \frac{y^2}{4}$

$1024x^5 - 640x^4y + 160x^3y^2$ ✓ A1

(b) Hence use the expansion in (a) above to **approximate** the value of $(39.6)^5$ (2mks)

$39.6 = (40 - 0.4)$ $\therefore 4x = 40$ & $\frac{y}{2} = 0.4$

$x = 10$ $y = 0.8$

$(1024 \times 10^5) - 640 \times 10^4 \times 0.8 + 160 \times 10^3 \times 0.8^2$ ✓ M1

$97,382,400$ ✓ A1

13. The cost per head for catering for a party is partly constant and partly varies inversely as the number of people expected. The cost per head for a party of 100 people is Sh. 1860 and that for 180 people is sh. 1060. **Find** the cost per head for 200 people. (3mks)

$C \propto k + \frac{1}{n}$

$C = k + \frac{m}{n}$ k & m constant

$1860 = k + \frac{m}{100}$
$1060 = k + \frac{m}{180}$ } M1

$186000 = 100k + m$
$190800 = 180k + m$
$-4800 = -80k$
$k = 60$ ✓ A1

$m = 186000 - 100 \times 60$
$= 180\,000$

$C = 60 + \frac{180000}{n}$

$C = 60 + \frac{180,000}{200}$

$= 960$ ✓ B1

14. A body is moving along a straight line and its acceleration after **t** seconds is (5 – 2t) ms^{-2}. Its initial velocity **V**ms^{-1} is $4ms^{-1}$. Find **V** in terms of **t**. (3 marks)

$V = \int (5-2t)\,dt$

$V = 5t - t^2 + c$ ✓M1

$4 = (5\times 0) - 0^2 + c$ ✓M1

$c = 4$

$V = 5t - t^2 + 4$ ✓A1

15. **Determine** the turning points for the curve y = 5x – 8x² + x³. (4mks)

$\frac{dy}{dx} = 5 - 16x + 3x^2$ ✓M1

$3x^2 - 16x + 5 = 0$ ✓M1

$(3x-1)(x-5) = 0$

$x = \frac{1}{3}$ & $x = 5$ ✓A1 (for both)

when $x = \frac{1}{3}$

$y = (5\times\frac{1}{3}) - 8\times(\frac{1}{3})^2 + (\frac{1}{3})^3$

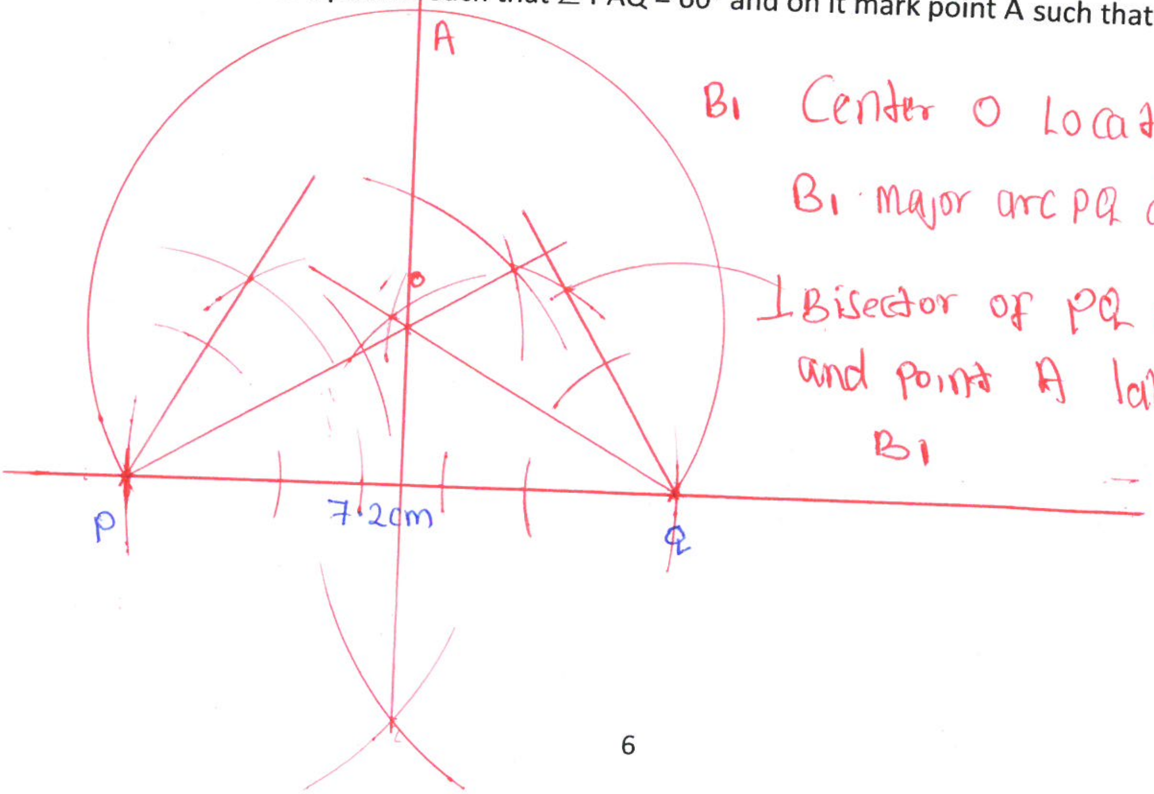
$y = \frac{22}{27}$

when $x = 5$

$y = -50$

$(\frac{1}{3}, \frac{22}{27})$ $(5, -50)$ B1 for both

16. Draw a line PQ = 7.2cm and on one side of the line, use a ruler and pair of compasses only to draw the locus of a point A such that ∠ PAQ = 60⁰ and on it mark point A such that PA = QA (3mks)



B1 Center O located ✓

B1 major arc PQ drawn ✓

⊥ Bisector of PQ drawn and point A labelled B1

SECTION II (50 Mks)

(*Answer* any FIVE *questions from this section*)

17. The table below represents marks scored in a mathematics test.

Marks	10-19	20-29	30-39	40-49	50-59	60-69	70-79
No. of students	2	6	7	13	6	4	2

Using an assumed mean of **44.5**, Determine

i) Mean marks for the test (3mks)

class	x	F	$t=x-A$	Ft	t^2	Ft^2	cf
10-19	14.5	2	-30	-60	900	1800	2
20-29	24.5	6	-20	-120	400	2400	8
30-39	34.5	7	-10	-70	100	700	15
40-49	44.5	13	0	0	0	0	
50-59	54.5	6	10	60	100	600	
60-69	64.5	4	20	80	400	1600	
70-79	74.5	2	30	60	900	1800	
		$\Sigma f = 40$		$\Sigma ft = -50$		Σft^2 8900	

Column of Ft ✓B1

$\bar{t} = \frac{\Sigma Ft}{\Sigma F} = \frac{-50}{40}$ ✓M1

$\bar{t} = -1.25$

$\bar{x} = 44.5 - 1.25$

$= 43.25$ ✓A1

ii) Standard deviation (4mks)

Column of Ft^2 ✓B1 any 4 correct

$\Sigma ft^2 = 8900$ ✓B1

$\sqrt{\frac{8900}{40} - \left(-1.25\right)^2}$ ✓M1

$= 14.86$ ✓A1

iii) Determine the pass mark if 30% of the students failed the exam. (3mks)

$\frac{30}{100} \times 40 = 12$ failed.

Pass mark ⇒ 13^{th} std

cf up to 3^{rd} class ✓B1

$29.5 + \left(\frac{13-8}{7}\right)10$ ✓M1

$29.5 + 7.143$

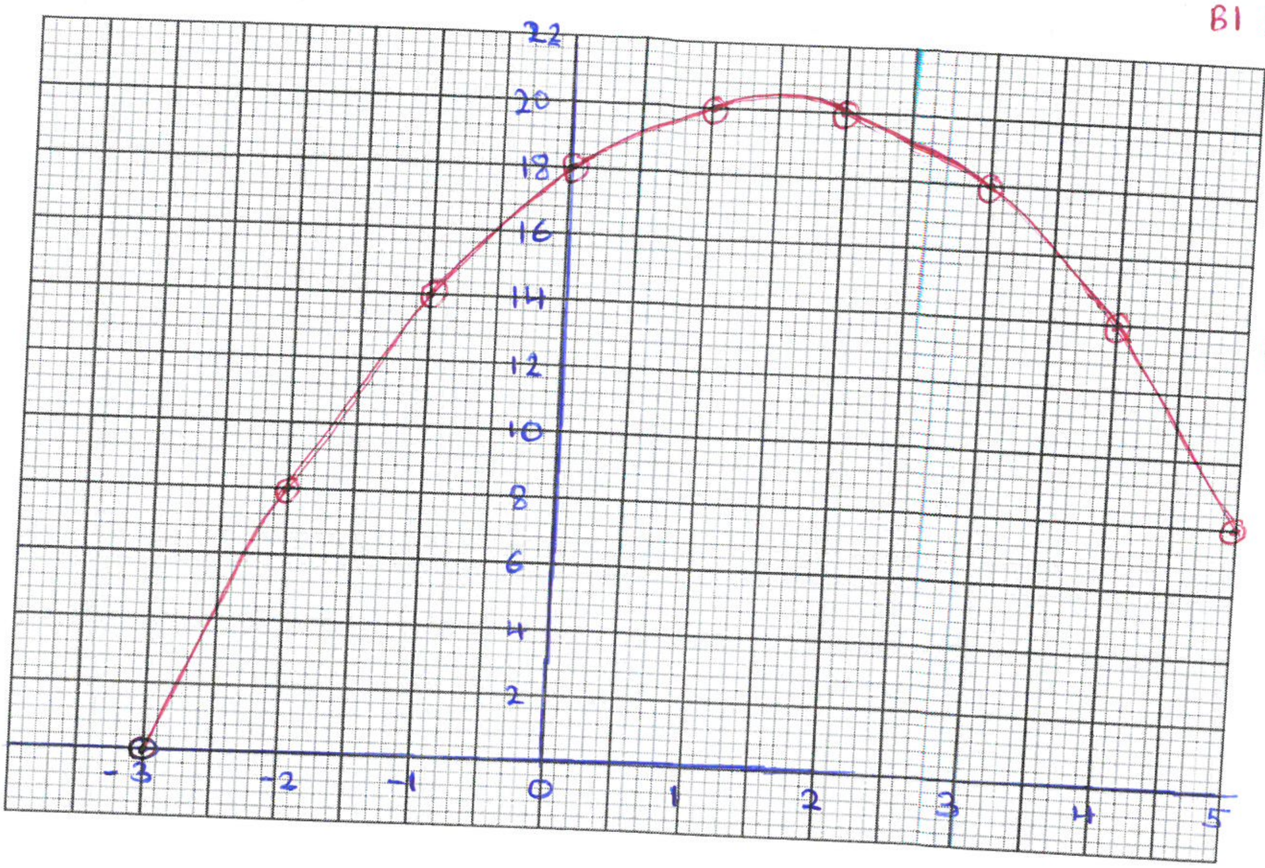
$= 36.64$ ✓A1

18. (a) Draw the curve of the function $y = 18 + 3x - x^2$ for $-3 \le x \le 5$. (3 marks)

Use a scale of 2cm to represent 1 unit on x axis and 1cm to represent 2 unit on y axis.

x	-3	-2	-1	0	1	2	3	4	5
y	0	8	14	18	20	20	18	14	18

B1 all co



P1

C1

(b) Find the actual area bounded by the curve, the x-axis and the line X= 5 (2 mks)

$$\int_{-3}^{5} (18 + 3x - x^2)\,dx$$

$$\left[18x + \frac{3}{2}x^2 - \frac{x^3}{3}\right]_{-3}^{5} \quad \text{M1}$$

$$\left[(18\times5) + \left(\frac{3}{2}\times5^2\right) - \frac{5^3}{3}\right] - \left[(18\times-3) + \left(\frac{3}{2}\times(-3)^2\right) - \frac{(-3)^3}{3}\right]$$

$$\frac{515}{6} - -\frac{63}{2} = 117\frac{1}{3} \quad \text{A1}$$

(c) By using trapezoidal rule with five ordinates, Estimate the area bounded by the curve, the x-axis and the line X= 5.

(3mks)

~~5~~ 5 ordinates $\Rightarrow$ 4 trapezia.

$h = \frac{5 - -3}{4} = \underline{\underline{2}}$ ✓ B1

$A = \frac{1}{2} \times 2 \{0 + 8 + 2(14 + 20 + 18)\}$ ✓ M1

$A = 112$ sq units ✓ A1

(d) Find the error introduced by the approximation.

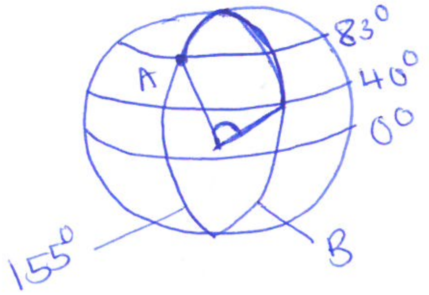
(2mks)

$\frac{117\frac{1}{3} - 112}{117\frac{1}{3}} \times 100$ ✓ M1

$= 4.545\%$ ✓ A1

19. An airplane leaves town **A** (83°N, 155°W) to town **B** (40°N, 25°E) using the shortest route at a speed of 450 knots. (Take **π** = $\frac{22}{7}$ and radius of the earth **R** = 6370km).

(a) (i) Calculate the distance between **A** and **B** in nautical miles. (2mks)



$155 + 25 = 180^0$

Thro North pole

$\theta = 180 - (83+40) = 57^0$ ✓B1

$60 \times 57 = 3420$ nm ✓B1

(ii) Calculate the time taken to travel from town **A** to **B** (2mks)

$\text{time} = \frac{3420}{450}$ ✓M1

$= 7.6$ hours ✓A1

or 7 hours 36 mins

(b) From **B**, the plane flies westwards along the latitude to town **C** (40°N, 13°W). Calculate the distance **BC** in kilometers. (3mks)

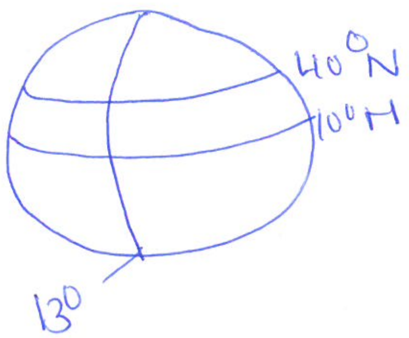
$(40^0 N, 25^0 E)$ $(40^0 N, 13^0 W)$

$\alpha = 25 + 15 = 40^0$ ✓B1

$\frac{40}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 40$ ✓M1

$= 3408.05$ km ✓A1

(c) From town **C**, the plane took off at 3:10 pm towards town **D** (10°N, 13°W), at the same speed. At what time did the plane land at **D**? (3mks)



$40 - 10 = 30^0$

$60 \times 30 = 1800$ nm ✓B1

$\text{time} = \frac{1800}{450}$ ✓M1 $= 4$ hours

3.10
4
7.10 pm ✓A1

20. The matrix $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$ represent a transformation T, triangle ABC where A(1,1) B(5,1) and C(2,4) is mapped onto $A^1B^1C^1$ by T.

a) i) Find the coordinates of the image $A^1B^1C^1$ of ABC under T. (2mks)

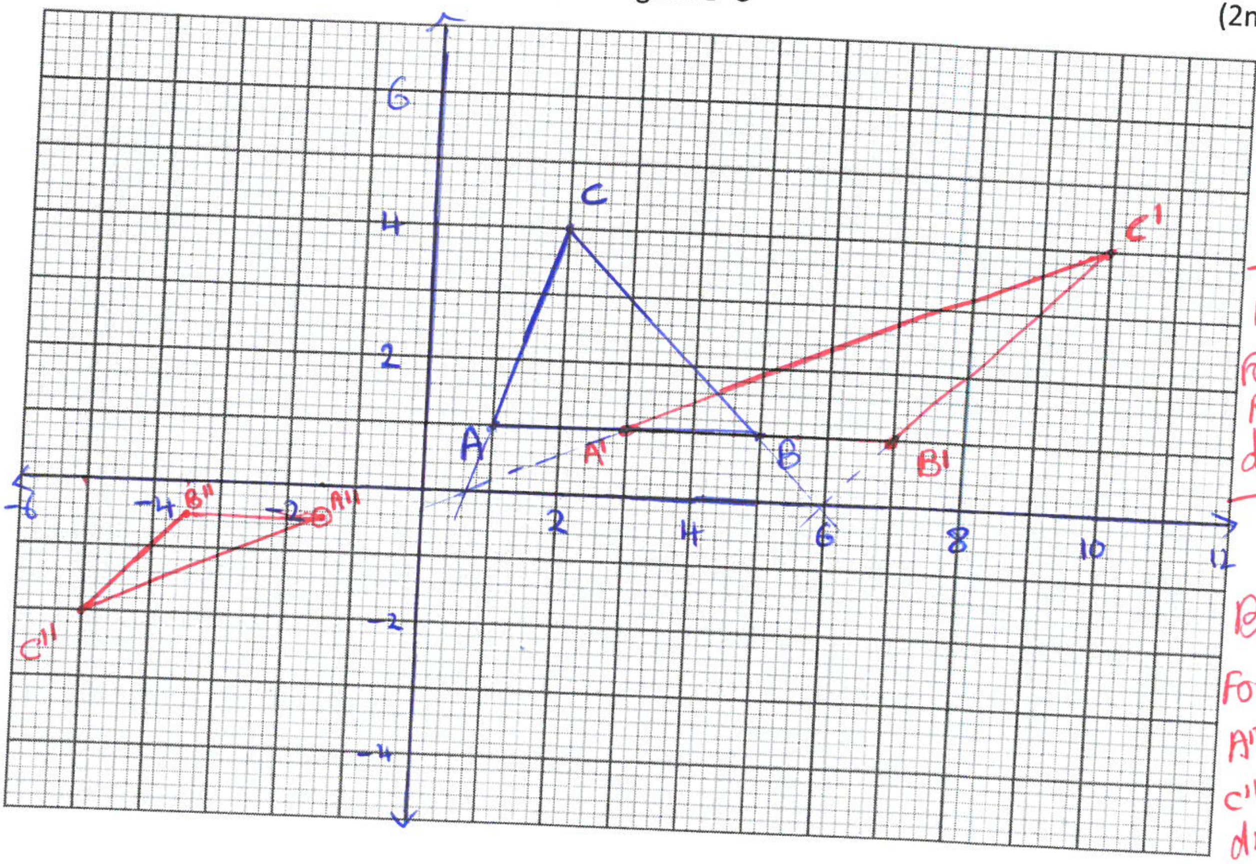
$$\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{array}{c} \begin{matrix}A & B & C\end{matrix}\\ \begin{bmatrix}1 & 5 & 2\\1 & 1 & 4\end{bmatrix}\end{array} = \begin{array}{c} \begin{matrix}A' & B' & C'\end{matrix}\\ \begin{bmatrix}3 & 7 & 10\\1 & 1 & 4\end{bmatrix}\end{array}$$

M1 Any two correct elements

A1 for

$A'(3,1)$ $B'(7,1)$ $C'(10,4)$

ii) On the grid provided draw ABC and its image $A^1B^1C^1$ (2mks)



B1 for ABC

B1 for A'B'C' drawn

B1 for A''B'' C'' drawn

iii) Describe the transformation T (1mk)

fully described shear

A Shear x- axis invariant and $C(2,4) \longrightarrow C'(10,4)$ B1

b) Draw $A^2B^2C^2$ image $A^1B^1C^1$ under an enlargement center (0, 0) scale factor - ½ (2 mks)

$$\begin{bmatrix}-\frac{1}{2} & 0\\0 & -\frac{1}{2}\end{bmatrix}\begin{bmatrix}3 & 7 & 10\\1 & 1 & 4\end{bmatrix} = \begin{bmatrix}-1.5 & -3.5 & -5\\-0.5 & -0.5 & -2\end{bmatrix}$$ B1 [OR By Construction]

c) Find a single matrix that would $A^2B^2C^2$ onto ABC. (3 mks)

$$\begin{bmatrix}-\frac{1}{2} & 0\\0 & -\frac{1}{2}\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} = \begin{bmatrix}-\frac{1}{2} & -1\\0 & -\frac{1}{2}\end{bmatrix}$$ B1

Inverse

$\det = \frac{1}{4} - 0 = \frac{1}{4}$

$4\begin{bmatrix}-\frac{1}{2} & -1\\0 & -\frac{1}{2}\end{bmatrix}$ M1

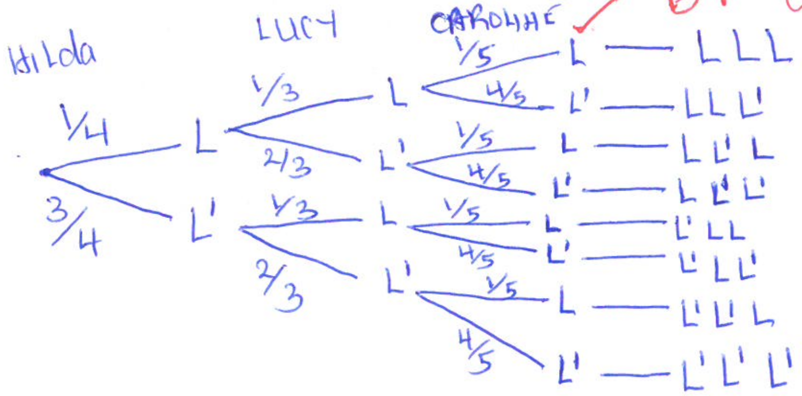
$= \begin{bmatrix}-2 & -4\\0 & -2\end{bmatrix}$ A1

21. The probability that Hilda, Lucy and Caroline will be late for breakfast on any one morning are $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{5}$ respectively.

a) Using a probability tree diagram find the probability that:-

i) None of them will be late (2mks)

B1 correct tree diagram



$P(L'L'L')$

$\frac{3}{4} \times \frac{2}{3} \times \frac{4}{5}$

$\frac{24}{60}$ B1

or $\frac{2}{5}$

(ii) Only one of them will be late (3mks)

$P(LL'L')$ or $P(L'LL')$ or $P(L'L'L)$

$\left[\frac{1}{4} \times \frac{2}{3} \times \frac{4}{5}\right] + \left[\frac{3}{4} \times \frac{1}{3} \times \frac{4}{5}\right] + \left[\frac{3}{4} \times \frac{2}{3} \times \frac{1}{5}\right] \rightarrow$ M1 any two correct brackets

$\frac{8}{60} + \frac{12}{60} + \frac{6}{60}$ M1 correct addition.

$= \frac{26}{60}$ A1

or $= \frac{13}{30}$

(iii) At least one of them will be late (3mks)

P(one late) or P(2 late) or P(all late) M1

$\frac{26}{60} + \left(\frac{1}{4} \times \frac{1}{3} \times \frac{4}{5}\right) + \left[\frac{1}{4} \times \frac{2}{3} \times \frac{1}{5}\right] + \left[\frac{3}{4} \times \frac{1}{3} \times \frac{1}{5}\right] + \left[\frac{1}{4} \times \frac{1}{3} \times \frac{1}{5}\right]$ M1

$\frac{26}{60} + \frac{4}{60} + \frac{2}{60} + \frac{3}{60} + \frac{1}{60}$

$= \frac{36}{60}$ A1

or $3/5$

OR

P(1 − all late)

$1 - \left[\frac{3}{4} \times \frac{2}{3} \times \frac{4}{5}\right]$

$1 - \frac{24}{60}$

$\frac{36}{60}$

(iv) At most one of them will be late (2mks)

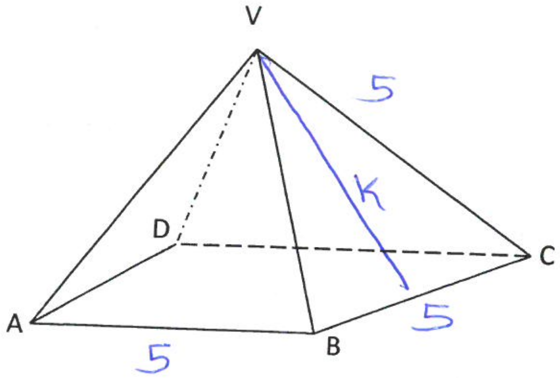
P(one late) or P(none)

$\frac{26}{60} + \left[\frac{24}{60}\right]$ M1

$= \frac{50}{60}$ A1

or $= \frac{5}{6}$

22. The figure below represents a square based pyramid with equilateral triangles AB=5cm



Calculate the

a) Height of the triangular faces (2mk)

$K = \sqrt{5^2 - 2.5^2}$ ✓M1

$= 4.330$ cm ✓A1

b) Length of AC (1mk)

$\sqrt{5^2 + 5^2}$

$= 7.071$ cm ✓B1

c) Angle between VA and ABCD (2mks)

$\cos\theta = \frac{3.536}{5}$ ✓M1

$\theta = \cos^{-1}\frac{3.536}{5}$

$\theta = 45^0$ ✓A1

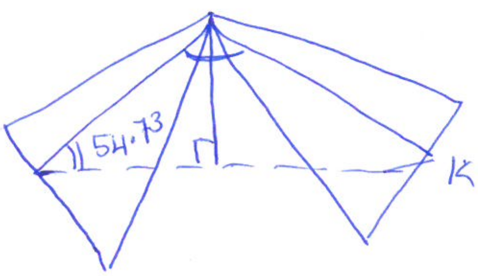
d) Angle between VAD and ABCD (2mks)

$\cos\theta = \frac{2.5}{4.33}$ ✓M1

$\theta = \cos^{-1}\frac{2.5}{4.33}$

$\theta = 54.73^0$ ✓A1

e) Angle between VAB and VCD (3mks)



$180 - (90 + 54.73)$ ✓M1

$= 35.27$ ✓A1

35.27×2

$= 70.54$ ✓B1

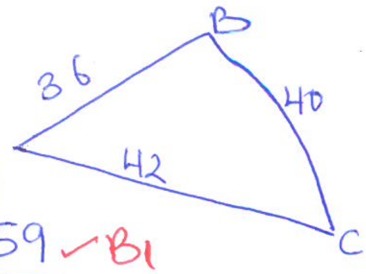
OR

$180 - (54.73 \times 2)$ ✓B2

$= 70.54$ ✓B1

23. A triangular plot ABC is such that AB = 36m, BC = 40cm and AC = 42m

(a) Calculate



(i) Area of the plot in square metres (3mks)

$S = \frac{1}{2}[36 + 40 + 42] = 59$ B1

$A = \sqrt{59(59-36)(59-40)(59-42)}$ M1

$A = \sqrt{59 \times 23 \times 19 \times 17} = 662.05\ m^2$ A1

(ii) Acute angle between the edges AB and BC (2mks)

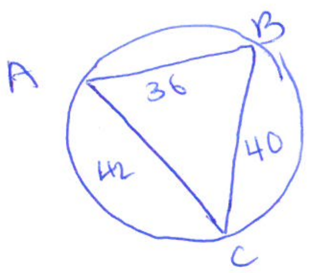
$\frac{1}{2} \times 36 \times 40 \sin B = 662.05$ M1

$\sin B = 0.9195$

$B = \sin^{-1} 0.9195 = 66.85^0$ A1

(b) A circular fence passes through vertices A, B and C. A water tap is to be installed inside the plot such that the tap is equidistant from each of the vertices A, B and C. Calculate

(i) The distance of the tap from vertex A (2mks)



$2R = \frac{42}{\sin 66.85^\circ}$ M1

$R = 22.84\ m$ A1

(ii) The area between the circular fence and the triangular plot (3mks)

$\pi r^2 - 662.05$

$\frac{22}{7} \times 22.84^2 = 1639.52$ M1

$A = 1639.52 - 662.05$ M1

$A = 977.47\ m^2$ A1

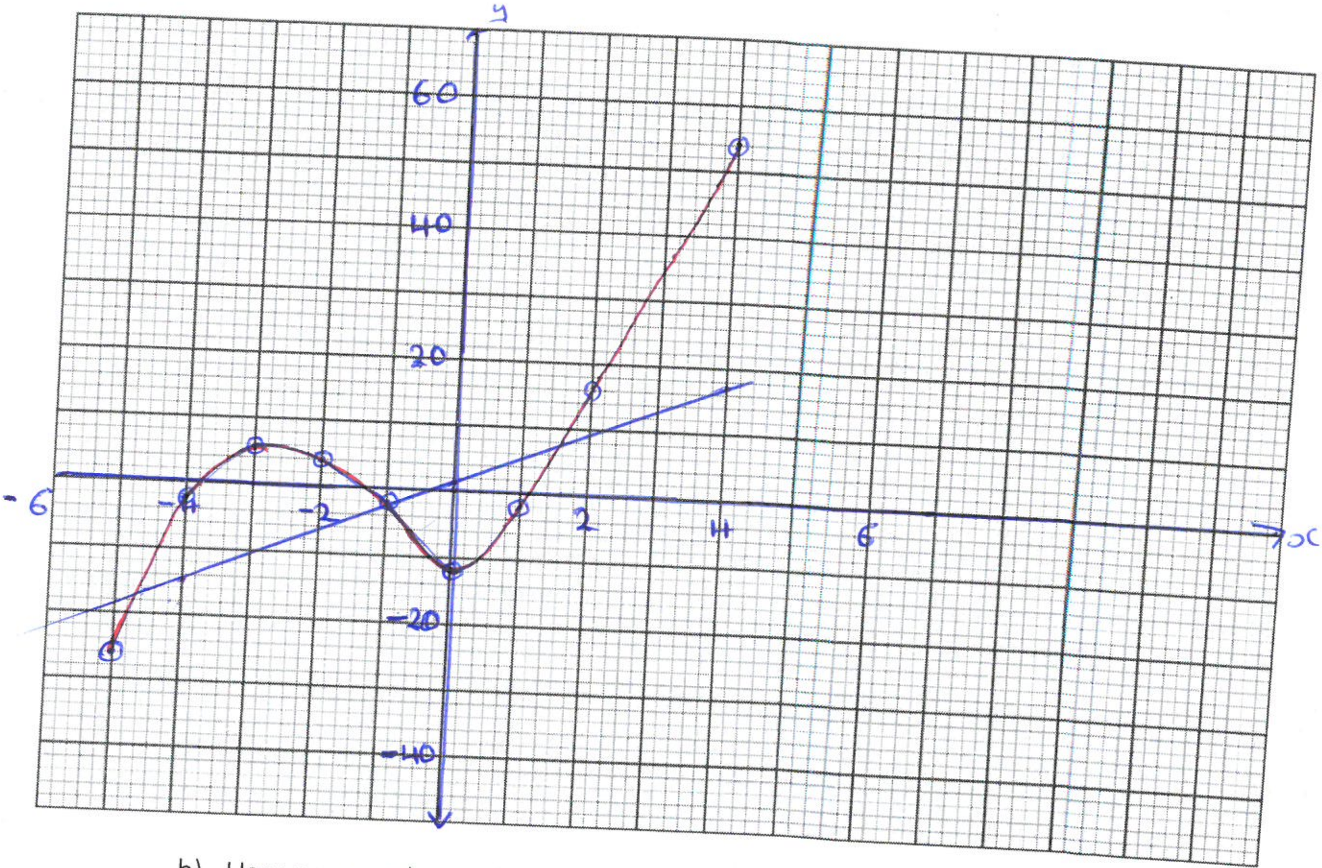
24. Fill the table below for the function $y=x^3+4x^2-x-6$ for $-5\leq x\leq 3$ (2marks)

X	-5	-4	-3	-2	-1	0	1	2	3
Y	-26	-2	6	4	-2	-6	-2	16	54

a) On the grid provided draw the graph of $y=x^3+4x^2-x-6$ for $-5\leq x\leq 3$. Use the scale of 1cm to represent 1 unit horizontally and 1cm to represent 10 units vertically. (3marks)

B1 Any ... Con...

B2 a...



S1

P1

C1

b) Use your graph to solve the following;

i. $y=x^3+4x^2-x-6=0$ (2marks)

$y = x^3 + 4x^2 - x - 6 = 0$

$0 = x^3 + 4x^2 - x - 6$

$y = 0$

$x = -3.8, \quad x = -1.3, \quad 1.1$

B2 for all Correct

B1 for any two

ii. $3x^3+12x^2-15x-21=0$ (3marks)

$y = 3x^3 + 12x^2 - 15x - 21$

$0 = x^3 + 4x^2 - 5x - 7$

$y = x^3 + 4x^2 - x - 6$

$0 = x^3 + 4x^2 - 5x - 7$

$y = 4x + 1$ B1

x	0	4	1
y	1	17	5

Line drawn = L1

$x = -4.7, -1, 1.6$ B1 for all