

MATHEMATICS ALT A



Dec. 2022 – 2½ hours

Name Index Number

Candidate's Signature Date

Instructions to candidates

- (a) Write your name and index number in the spaces provided above.
- (b) Sign and write the date of examination in the spaces provided above.
- (c) This paper consists of **two** sections: **Section I** and **Section II**.
- (d) Answer **all** the questions in **Section I** and only **five** questions from **Section II**.
- (e) **Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.**
- (f) Marks may be given for correct working even if the answer is wrong.
- (g) **Non-programmable** silent electronic calculators and KNEC Mathematical tables may be used except where stated otherwise.
- (h) **This paper consists of 19 printed pages.**
- (i) **Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**
- (j) **Candidates should answer the questions in English.**

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Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total

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SECTION I (50 marks)

Answer all the questions in this section in the spaces provided.

1. An investor took a loan from a bank that charged interest. The loan and the interest accrued were repaid in monthly instalments. The investor repaid Ksh 1 500 in the first month and in each subsequent month the instalments were reducing by Ksh 50 until the loan was fully repaid. Determine the maximum amount that may be paid for that loan. (3 marks)

2. Two machines A and B working independently can take 8 hours and 10 hours respectively to do a task. A third machine C and machine A working together can do the same task in 5 hours. Determine the time it would take machine B and machine C working together to do the same task. (3 marks)

3. Simplify $\frac{3 + \sqrt{5}}{7 - 3\sqrt{5}}$, leaving the answer in the form $a + b\sqrt{c}$ where a, b and c are integers. (2 marks)

4. The market value of a certain precious stone varies directly as the square of its mass. One such stone of mass 10 kg has a value of Ksh 600 000.

Calculate the value of a similar stone whose mass is 18.5 kg. (3 marks)

5. The perimeter of a rectangle is 48 cm while its area is 108 cm^2 . Form a quadratic equation to represent the situation and hence determine the dimensions of the rectangle. (3 marks)

6. Two parallel chords $AB = 4 \text{ cm}$ and $CD = 10 \text{ cm}$ lie on opposite sides of a centre O of a circle. The perpendicular distance between the two chords is 7 cm.

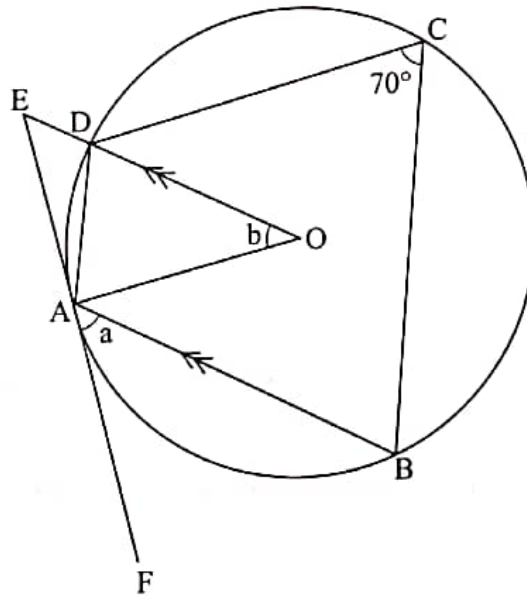
Calculate the radius of the circle leaving the answer in surd form. (3 marks)

7. A rectangle ABCD in which $AB = 12$ cm and $BC = 5$ cm is the base of a right pyramid whose apex is V. $VA = VB = VC = VD = 13$ cm. Point M is the mid point of the edge VC.

Calculate, correct to 2 decimal places, the length of line AM.

(3 marks)

8. In the figure below, O is the centre of the circle. Points A, B, C and D lie on the circumference of the circle. Line AB is parallel to the straight line EDO and line FAE is a tangent to the circle at A. $\angle FAB = a^\circ$, $\angle DOA = b^\circ$, $\angle DCB = 70^\circ$



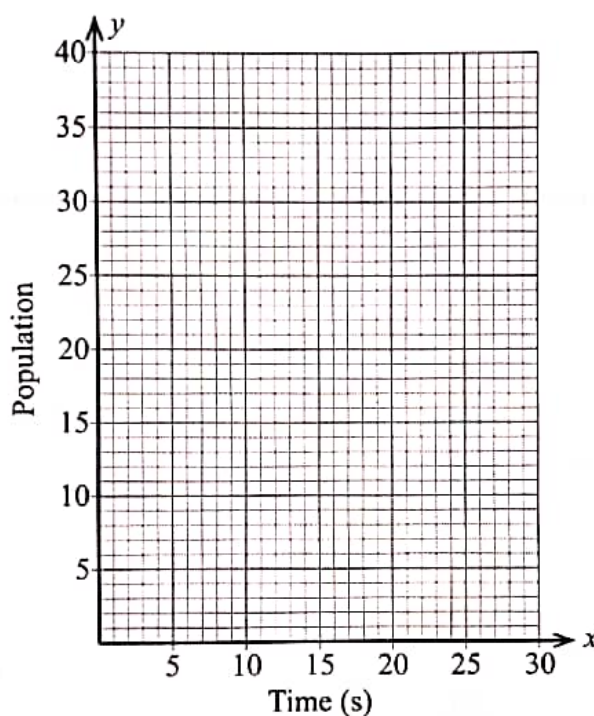
Determine the values of a and b.

(4 marks)

9. The population growth of a colony of bacteria was recorded at intervals of 5 seconds(s) as shown in the table below

t(s)	0	5	10	15	20	25
Number of bacteria	5	7	11	16	24	36

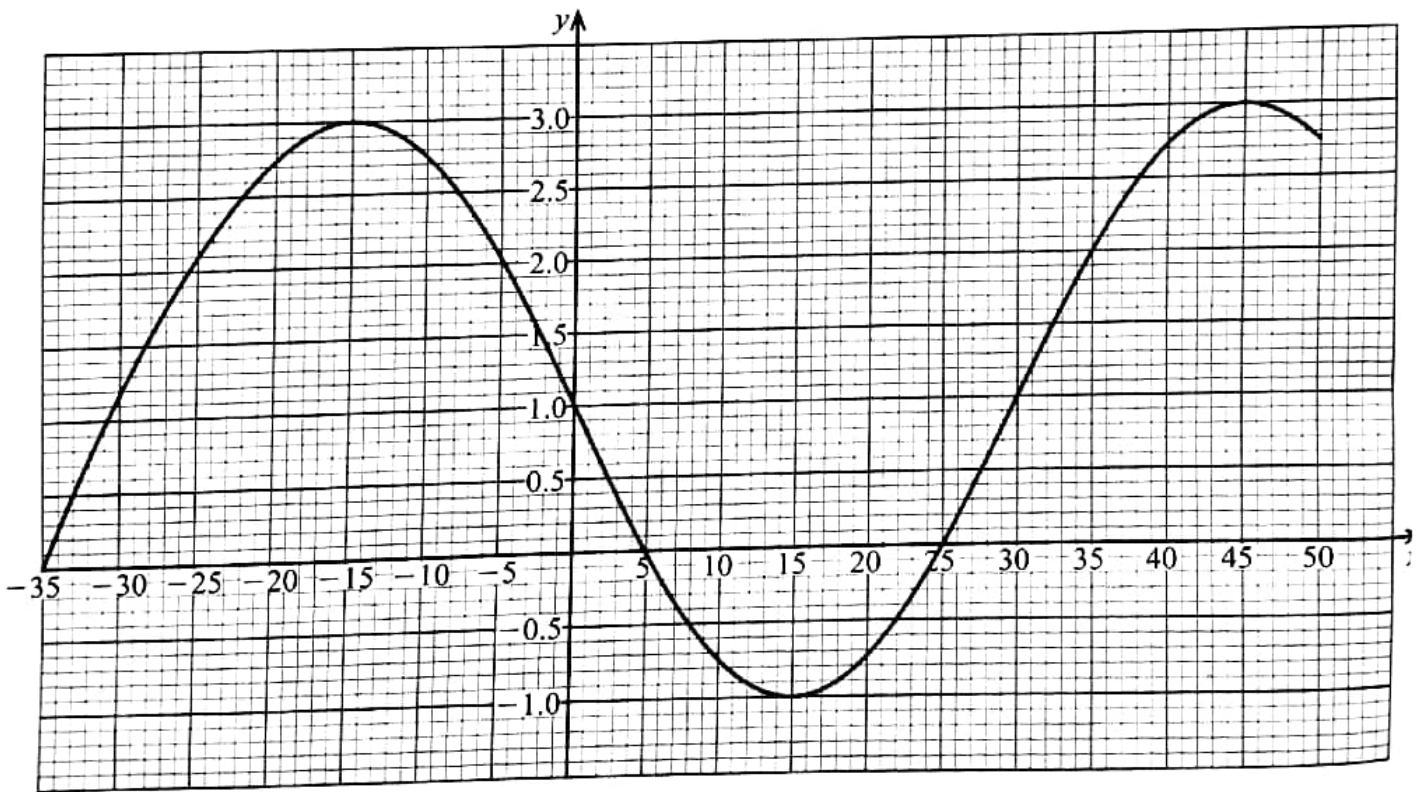
- (a) On the grid provided, draw a graph of the population of bacteria against time. (2 marks)



- (b) Use the graph to determine, correct to 2 decimal places, the average rate of change of the population of bacteria between $t = 5$ seconds and $t = 20$ seconds. (2 marks)

10. A circle centre $C(5, 5)$ passes through points $A(1, 3)$ and $B(a, 9)$. Find the equation of the circle and hence the possible values of a . (3 marks) 12.

11. The figure below represents the curve of the function $y = 1 - A \sin wx$ for the range $-35^\circ \leq x \leq 50^\circ$. 13.



Determine the values of A and w .

(3 marks) 14.

12. The data below represents the number of animals owned by 7 neighbours:

9, 5, 14, 6, 8, 13 and 15.

Calculate, correct to the nearest whole number, the standard deviation of the number of animals.
(3 marks)

13. The table below shows income tax rates in a certain year.

Monthly taxable income in Kenya shillings	Tax rates in each shilling (%)
0 – 12 298	10
12 299 – 23 885	15
23 886 – 35 472	20

A tax relief of Ksh 1 408 per month was allowed. Calculate the monthly income tax paid by an employee whose monthly taxable income was Ksh 26 545.75.
(3 marks)

14. Point P(8, 4, -1) divides line AB internally in the ratio 4 : 1. The position vector of point A with

respect to the origin O is $\begin{pmatrix} -4 \\ 8 \\ 3 \end{pmatrix}$. Determine the coordinates of point B. (3 marks)

15. An aircraft took off from an airport A(0° , 40° W) at 1100 h local time. The aircraft landed at airport B(0° , 65° W) at 1200 h local time.

Determine the speed of the aircraft in knots.

(4 marks)

16. The velocity v m/s of a particle moving in a straight line is $(-2t + 4)$ m/s. Determine the distance moved by the particle during the first second of its motion.

(3 marks)

SECTION II (50 marks)

Answer only five questions from this section in the spaces provided.

17. A wholesaler stocks two types of rice: Refu and Tamu. The wholesale prices of 1 kg of Refu and 1 kg of Tamu are Ksh 80 and Ksh 140 respectively. The wholesaler also stocks blend A rice which is a mixture of Refu and Tamu rice mixed in the ratio 3 : 2.

- (a) (i) A retailer bought 10 kg of blend A rice. To this blend, the retailer added some Tamu rice to prepare a new mixture blend X. The ratio of Refu rice to Tamu rice in blend X was 1 : 2.

Determine the amount of Tamu rice that was added. (3 marks)

- (ii) The retailer sold blend X rice making a profit of 20%. Determine the selling price of 1 kg of blend X. (3 marks)

- (b) The wholesaler prepared another mixture, blend B, by mixing x kg of blend A rice with y kg of Tamu rice. Blend B has a wholesale price of Ksh 130 per kg.

Determine the ratio $x : y$. (4 marks)

18. Two bags P and Q contain identical marbles except for the colours. Bag P contains 3 green and 4 red marbles. Bag Q contains 2 green and 3 red marbles.

(a) Find the probability of picking a red marble from bag P. (1 mark)

(b) Two marbles were picked at random from bag P, one at a time, without replacement.

(i) Draw a probability tree diagram to show all the possible outcomes. (1 mark)

(ii) Find the probability that the two marbles picked were of the same colour. (2 marks)

(iii) Find the probability that at least one red marble was picked. (2 marks)

(c) The marbles picked from bag P in (b) were both put into bag Q. A marble was then picked at random from bag Q.

Calculate the probability that the marble picked was:

(i) green in colour (3 marks)

(ii) red in colour (1 mark)

19. A transformation matrix $T_1 = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$ maps a triangle ABC onto triangle A'B'C'. Another transformation matrix $T_2 = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ maps triangle A'B'C' into triangle A''B''C''.

The coordinates of point C'' is (10, 8) and the area of triangle A''B''C'' is 15 square units.

- (a) (i) Determine the coordinates of C. (5 marks)

- (ii) Determine the area of triangle ABC. (3 marks)

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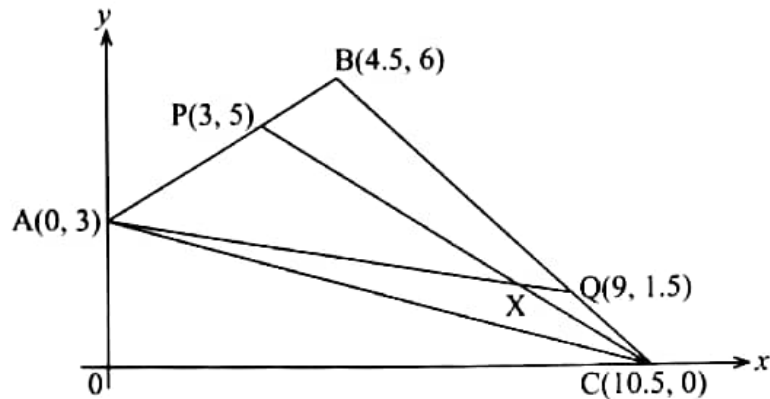
- (b) The coordinates of points B and B'' are (x, y) and (6x + 1, 8) respectively.

Determine the value of y. (2 marks)

c)

20. In the diagram below, the vertices of triangle ABC are $A(0, 3)$, $B(4.5, 6)$ and $C(10.5, 0)$.

Points $P(3, 5)$ and $Q(9, 1.5)$ lie on lines AB and BC respectively.



- (a) Find:
- (i) AQ (1 mark)
- (ii) CP (1 mark)
- (b) Lines AQ and CP intersect at X such that $CX = kCP$ and $AX = mAQ$ where k and m are scalars.
- (i) By expressing OX in two different ways, determine the values of k and m . (6 marks)
- (ii) Determine the exact coordinates of point X . (2 marks)

- (a) Juma bought a house 4 years ago for Ksh 2 500 000. The value of the house rose steadily over 4 years to its current value of Ksh 3 700 000.

Calculate, correct to 2 decimal places, the annual rate of appreciation in the value of the house. (3 marks)

- (b) At the time Juma bought the house in 21(a), Tony also bought a car valued at Ksh 5 100 000. The value of the car depreciated steadily at a rate of 2% every 4 months.

Determine correct to the nearest shilling, the current value of the car. (3 marks)

- (c) The house bought in 21(a) continued to appreciate in value at the same rate while the car bought in 21(b) continued to depreciate in value at the same rate. Determine the number of years from the time of purchase, it would take for the value of the house and that of the car to be equal. Give the answer correct to 1 decimal place. (4 marks)

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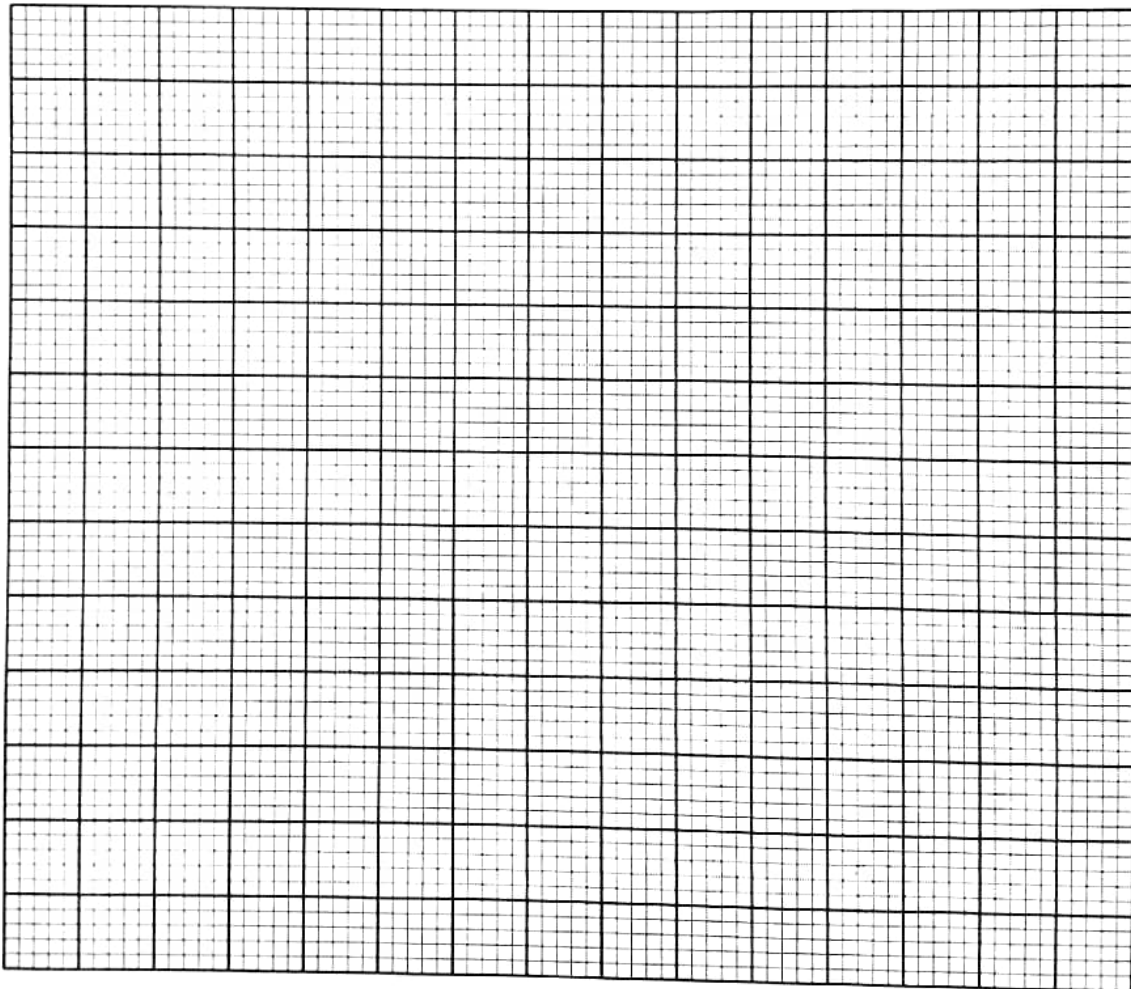
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22. Fifty teachers in a sub county attended a workshop. The table below shows the distribution of the distances (d) in kilometres travelled by the teachers from their respective school to the training venue.

Distance d (km)	0 – 4	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29
No. of teachers	4	7	11	14	9	5

- (a) On the grid provided, draw a cumulative frequency graph to represent the information above. (4 marks)



(b) Use the graph to estimate:

(i) the median distance (1 mark)

(ii) the number of teachers who travelled a distance d km where $15 \leq d \leq 23$ (3 marks)

(c) Each of the 75% of all the teachers who travelled a distance d km where $d \leq 10$ km, used a motor bike and each was charged Ksh 50.

Determine the total amount of money raised by the motor bike operators. (2 marks)

23. In an inter school mathematics contest, schools can register teams in junior and senior categories. Information on number of students and the participation fee per team in each category is given in the table below.

	Junior category	Senior category
No. of students per team	6	4
Participation fees per team	Ksh 2 000	Ksh 3 000

The organising committee projected to register x junior teams and y senior teams.

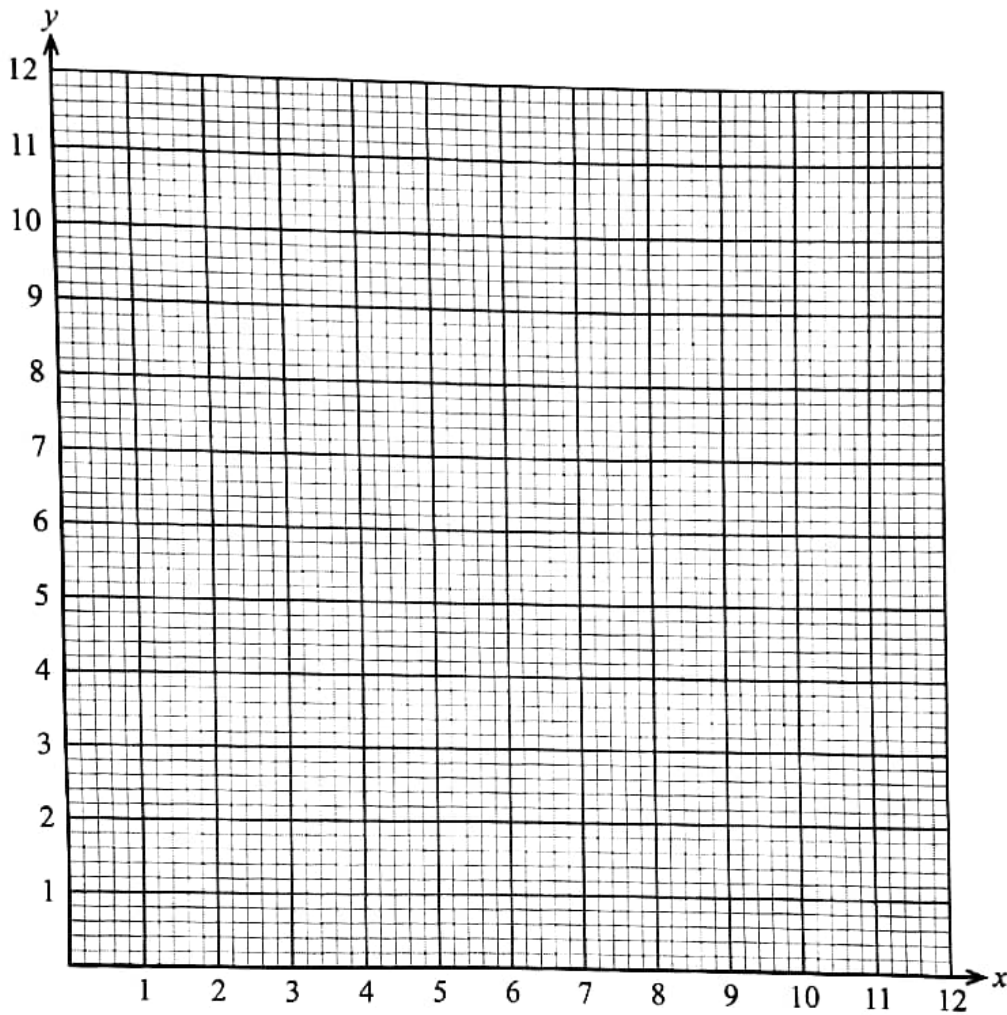
- (a) For the contest to take place, the following conditions must be satisfied:
- (i) At least two junior teams must be registered
 - (ii) The number of senior teams must be more than half the number of junior teams
 - (iii) The total number of participating students from the two categories must not exceed 48
 - (iv) The total amount of money raised from the participation fees must be more than Ksh 12,000

Write down inequalities in x and y that satisfy the conditions.

(4 marks)

(b) Represent the inequalities in (a) on the grid provided.

(4 marks)



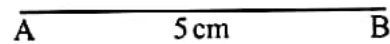
(c) The organising committee expected to make a profit of Ksh 200 for every junior team and Ksh 500 for every senior team that participated.

Determine the number of teams of each category that should be registered in order to maximise the profit.

(2 marks)

24. In this question use a ruler and a pair of compasses.

The line AB drawn below is a side of triangle ABC in which $\angle ABC = 90^\circ$ and $\angle BAC = 60^\circ$.



- (a) Complete triangle ABC. (2 marks)
- (b) Construct the locus of points P such that $\angle APB = 30^\circ$. (2 marks)
- (c) Locate by construction points Q_1 and Q_2 which satisfy the conditions below.
- (i) Q_1 and Q_2 lie on the same side of line AB as C.
- (ii) Area of $\triangle AQ_1B = \text{Area of } \triangle AQ_2B = \frac{3}{4} \text{ Area of } \triangle ABC$.
- (iii) $\angle AQ_1B = \angle AQ_2B = 30^\circ$. Measure the length of line Q_1Q_2 . (3 marks)

- (d) Calculate the area above line Q_1Q_2 bounded by the locus of points P. (3 marks)

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Kenya Certificate of Secondary Education, 2022
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