Contents

1

KCSE 2022

KCSE 2022 Paper 1

Problem 1 ■■■ KCSE2022/P1/No. 1

Solve for n

$$\frac{6n}{n-1} = \frac{25}{n}$$

(**3 marks**)

SOLUTION:

$$\frac{6n}{n-1} = \frac{25}{n}$$

$$6n^2 = 25(n-1) = 25n - 1 \checkmark M_1$$

$$6n^2 - 25n + 1 = 0$$

$$(3n-5)(2n-5) = 0 \qquad \checkmark M_1$$

$$3n = 5 \implies n = \frac{5}{3} = 1\frac{2}{3}$$

$$2n = 5 \implies n = \frac{5}{2} = 2\frac{1}{2} \qquad \checkmark A_1$$

Problem 2 ■■■ KCSE2022/P1/No. 2

A family used two-fifths of its monthly income on school fees. Three-quarters of the remaining amount was used on family upkeep while the rest was invested. the family invested **Ksh. 13500** monthly.

Calculate the amount of money the family used on school fees every month. (**4 marks**)

SOLUTION:

Let x = the total monthly income.

$$\text{school fees} = \frac{2}{5}x$$

$$\text{upkeep} = \frac{3}{4} \text{ of } \left(x - \frac{2}{5}x\right) = \frac{9}{20}x$$

$$\text{investment} = x - \left(\frac{2}{5}x - \frac{9}{20}x\right) = \frac{3}{20}x = \text{Ksh. } 13500 \quad ✓M_1$$

$$\implies x = 13500 \times \frac{20}{3} = 90000$$

$$\text{school fees} = \frac{2}{5}x = \frac{2}{5} \text{ of } 90000 \quad ✓M_1$$

$$= \text{Ksh. } 36000 \quad ✓A_1$$

Problem 3

KCSE2022/P1/No. 3

Solve for x in the equation.

$5^{2x-1} - 25^x = 500$ (**3 marks**)

SOLUTION:

$$5^{2x-1} - 25^x = 500$$

$$\frac{1}{5} \cdot 5^{2x} - 5^{2x} = 500 \quad ✓M_1$$

$$5^{2x}\left(\frac{1}{5} - 1\right) = 500 \qquad \text{factor out } 5^{2x}$$

$$5^{2x} \cdot -\frac{4}{5} = 500 \quad ✓M_1$$

$$\implies 5^{2x} = -625 = -5^4$$

$\therefore$ no real x solution, since 5^{2x} cannot be negative. $✓A_1$

NOTE

For all $a > 0$, a^k is always positive for every integer k.

Problem 4

KCSE2022/P1/No. 4

Kipkoech and Tanui began a **5000 m** race together at the starting line. Kipkoech and Tanui took **72** seconds and **80** seconds respectively to run a **400 m** lap. The two athletes were together again at the starting line after some time.

Determine the number of laps that Tanui had to run to complete the race after they were together. (**3 marks**)

SOLUTION:

$$72 = 2^3 \times 3^2$$

$$80 = 2^4 \times 5$$

$$\mathbf{lcm}(72, 80) = 2^4 \times 3^2 \times 5 = 720 \quad ✓M_1$$

$$\text{Laps made by Tanui} = \frac{720}{80} = 9 \quad ✓M_1$$

$$\text{Remaining laps} = \frac{5000}{400} - 9 = 3\frac{1}{2} \text{ laps} \quad ✓A_1$$

Problem 5 KCSE2022/P1/No. 5

Simplify

$$\frac{18ax-(3a-4x)(3a+4x)}{3a-8x}$$ (3 marks)

SOLUTION:

$$\frac{18ax-(3a-4x)(3a+4x)}{3a-8x}=\frac{18ax-(9a^2-16x^2)}{3a-8x}$$

$$=\frac{16x^2+18ax-9a^2}{3a-8x}$$ $\checkmark M_1$ expand the brackets

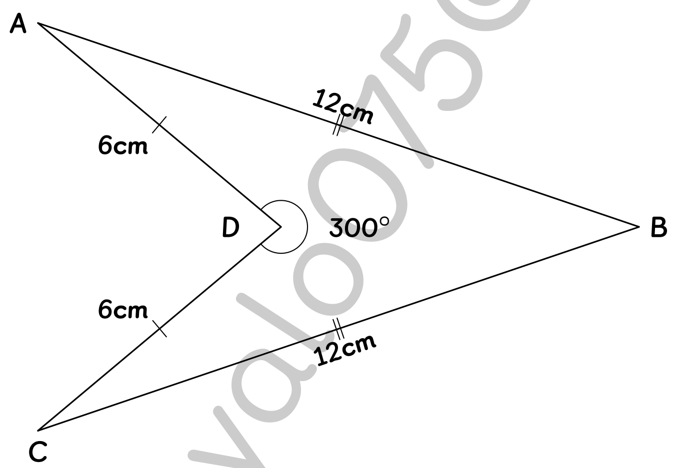
$$=\frac{16x^2+24ax-6ax-9a^2}{3a-8x}$$

$$=\frac{(2x+3a)\cancel{(8x-3a)}}{-1\cancel{(8x-3a)}}$$ $\checkmark M_1$ factorise and simplify

$$=-1(2x+3a)=-2x-3a$$ $\checkmark A_1$

Problem 6 KCSE2022/P1/No. 6

In the quadrilateral ABCD, AD = CD = CD = **6cm** and BA = BC = **12cm**. Angle ADC = 300°.



Calculate, correct to 2 decimal places, the area of the quadrilateral ABCD. (4 marks)

SOLUTION:

Area of $\triangle ADC = \frac{1}{2}\times 6\times 6\sin 60^\circ = 9\sqrt{3}\text{cm}^2$ $\checkmark M_1$

Area of $\triangle ABC = \sqrt{15(15-12)(15-12)(15-6)}$ $\checkmark M_1$ $AC = 6\text{cm},\ s = 15$ since $\triangle ACD$ is equilateral

$=9\sqrt{15}\text{cm}^2$

Area of ABCD $=9\sqrt{15}-9\sqrt{3}=19.2683$ $\checkmark M_1$ use a calculator

$\approx 19.27\text{cm}^2$ $\checkmark A_1$

KEYPOINT

Area of a triangle:

$\frac{1}{2}ab\sin C$

OR

$\sqrt{s(s-a)(s-b)(s-c)}$

where $s=\frac{a+b+c}{2}$

Problem 7

KCSE2022/P1/No. 7

A watch loses **8** seconds every hour. It was set to read the correct time at **1100h** on Sunday.

Determine the time, in a **12**-hour system, the watch will show on the following Thursday when the correct time is **0500h**. (**3 marks**)

SOLUTION:

$$\textbf{1100h} \text{ Sunday to } \textbf{1100h} \text{ Wednesday } = 24 \times 3 = 72\text{h}$$

$$\textbf{1100h} \text{ Wednesday to } \textbf{0500h} \text{ Thursday } = 18\text{h}$$

$$\text{Total time } = 18 + 72 = 90\text{h}$$

$$\text{Time lost } = \frac{90 \times 8}{60} = 12 \text{ min} \quad \checkmark M_1$$

$$\text{Time on Thursday } = 5 : 50 \text{ am} - 12 \text{ min} \quad \checkmark M_1$$

$$= 4 : 48 \text{ am} \quad \checkmark A_1$$

Problem 8

KCSE2022/P1/No. 8

A lorry left town **A** for town **B** and maintained an average speed of **50km/h**. A car left town **A** for town **B** **42** minutes later and maintained an average speed of **80km/h**. At the time the car arrived in town **B**, the lorry had **25km** to cover to town **B**.

Determine the distance between town **A** and **B**. (**3 marks**)

SOLUTION:

Let **t** = time taken by car from town **A** to town **B**.

	Speed(km/h)	Time(h)	Distance(km)
Lorry	50	$\frac{42}{60} + t$	$50\left(t + \frac{42}{60}\right)$
Car	80	t	80t

The Lorry had **25km** more to cover when car arrived at town **B**, thus:

$$80t = 50\left(t + \frac{42}{60}\right) + 25 \quad \checkmark M_1$$

$$80t = 50t + 35 + 25$$

$$30t = 60 \implies t = 2\text{h} \quad \checkmark M_1$$

hence the distance between the two towns is:

$$= 80t = 80 \times 2 = 160\text{km} \quad \checkmark A_1$$

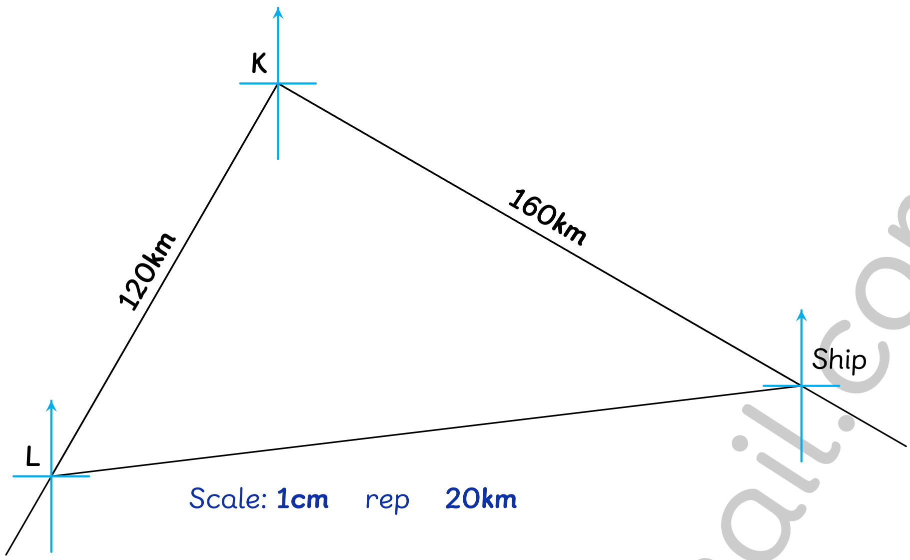
Problem 9

KCSE2022/P1/No. 9

Port **L** is **120km** on a bearing of **S30°W** from port **K**. A ship left port **K** at **1000h** and sailed at a speed of **40km/h** along the bearing of **S60°E**.

Using scale drawing, determine the bearing of the ship from port **L** at **1400h**. (**4 marks**)

SOLUTION:



Distance covered $= 40 \times 4 = 160$km　　Distance = Speed × Time

Bearing $=$ N83°E　　Compass bearing

$= 083°$ ✓B_1　　True bearing

SCALE DRAWING MARKING RUBRIK	
Accurate scale drawing	✓S_1
Position of L	✓B_1
Position of Ship	✓B_1

Problem 10 KCSE2022/P1/No. 10

The image of $P(-2, 5)$ under a translation T is $P'(2, 2)$. $Q'(9, -5)$ is the image of Q under the same translation T.

Determine the coordinates of Q. (**3 marks**)

SOLUTION:

$$T + \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\Longrightarrow T = \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad ✓M_1$$

$$T + \overrightarrow{OQ} = \overrightarrow{OQ'}$$

$$\begin{pmatrix} 4 \\ -3 \end{pmatrix} + \overrightarrow{OQ} = \begin{pmatrix} 9 \\ -5 \end{pmatrix}$$

$$\Longrightarrow \overrightarrow{OQ} = \begin{pmatrix} 9 \\ -5 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \quad ✓M_1$$

$$\therefore Q(5, -2) \quad ✓A_1$$

Problem 11 KCSE2022/P1/No. 11

A Kenyan bank bought and sold United Arab Emirates (UAE) dirhams on two different dates as shown below.

		Buying(Ksh.)	Selling(Ksh.)
1st August 2021	1 UAE dirham	28.40	28.90
16th August 2021	1 UAE dirham	28.00	28.40

A Kenyan tourist who travelled to UAE on 1st August 2021 converted **Ksh. 130050** to UAE dirhams.

During her stay in UAE, she spent **3520** UAE dirhams. She arrived back to Kenya on 16th August 2021. On the same day she converted the remaining amount of money to Kenya shillings at the same bank.

Calculate the amount of money in Kenya shillings that she received from the bank. (**3 marks**)

SOLUTION:

$$\text{Dirhams received} = \frac{130050}{28.90} = 4500 \quad \checkmark M_1$$ use a

$$\text{Remainder after expenses} = 4500 - 3520 = 980$$

$$\textbf{Ksh.}\text{ received} = 980 \times 28.00 \quad \checkmark M_1$$

$$= \textbf{Ksh. 27440} \quad \checkmark A_1$$

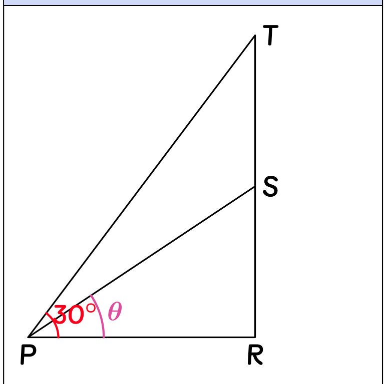
Problem 12

KCSE2022/P1/No. 12

An electric post erected vertically is **20 m** from point **P** on the same level ground. The angle of elevation of the top, **T**, of the post from **P** is 30°. Given that **S** is the mid point of the post, calculate, correct to **1** decimal place, the angle of elevation of **S** from **P**. (**3 marks**)

SOLUTION:

SKETCH



$$RT = 20\tan 30$$

$$RS = \frac{1}{2}RT = \frac{1}{2} \times 20\tan 30 = 10\tan 30^\circ \quad \checkmark M_1$$

$$\tan\theta = \frac{SR}{PR} \Longrightarrow \theta = \tan^{-1}\frac{SR}{PR}$$

$$\theta = \tan^{-1}\left(\frac{10\tan 30}{20}\right) = \tan^{-1}(0.2887) \quad \checkmark M_1$$ use a

$$= 16.1^\circ \quad \checkmark A_1$$ use a

Problem 13

KCSE2022/P1/No. 13

Given that $A = \begin{pmatrix} 2 & 6 \\ 2u & 5 \end{pmatrix}$, $B = \begin{pmatrix} 7 & -3 \\ -u & 5 \end{pmatrix}$ and $BA = \begin{pmatrix} 2 & v \\ 16 & w \end{pmatrix}$, determine the values of u, v and w. (**3 marks**)

SOLUTION:

$$BA = \begin{pmatrix} 7 & -3 \\ -u & 5 \end{pmatrix}\begin{pmatrix} 2 & 6 \\ 2u & 5 \end{pmatrix} = \begin{pmatrix} 14-6u & 27 \\ 8u & 25-6u \end{pmatrix} \quad \checkmark M_1$$

$$\therefore \begin{pmatrix} 14-6u & 27 \\ 8u & 25-6u \end{pmatrix} = \begin{pmatrix} 2 & v \\ 16 & w \end{pmatrix}$$

$$\Longrightarrow 8u = 16 \Longrightarrow u = 2 \quad \checkmark A_1$$ equate corresponding terms

$$\implies v = 27$$

$$\implies 25 - 6(2) = 3 \implies w = 13 \quad \checkmark A_1$$

Problem 14 KCSE2022/P1/No. 14

The capacities of two similar containers are **54ml** and **250ml** respectively. The difference in the heights of the two containers is **4cm**.

Calculate the height of the larger container. (**3 marks**)

SOLUTION:

Let **x** = height of larger container.

$$\frac{\text{volume of larger container}}{\text{volume of smaller container}} = \frac{250}{54} = \frac{125}{27} \quad \checkmark M_1$$

$$\frac{\text{height of larger container}}{\text{height of smaller container}} = \left(\frac{125}{27}\right)^{\frac{1}{3}} = \frac{5}{3}$$

$$\frac{x}{x-4} = \frac{5}{3} \quad \checkmark M_1$$

$$3x = 5x - 20 \implies x = 10$$

$\therefore$ Height of larger container is **10cm** $\checkmark A_1$

Problem 15 KCSE2022/P1/No. 15

The table below shows the mean marks in a mathematics test of two classes.

Class	Number of students	Mean mark
X	43	65
y	45	62

Calculate, correct to 2 decimal places, the mean mark of the classes. (**2 marks**)

SOLUTION:

$$\text{Mean} = \frac{(43 \times 65) + (45 \times 62)}{43 + 45} \quad \checkmark M_1$$

$$= 63.47 \quad \checkmark A_1$$

use a

KEYPOINT

Mean of combined series is:

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Problem 16 KCSE2022/P1/No. 16

The base, **ABCDEF**, of a right pyramid is a regular hexagon of side **2.5cm**. Point **V** is the vertex of the pyramid and the length of the slanting edges is **4cm**.

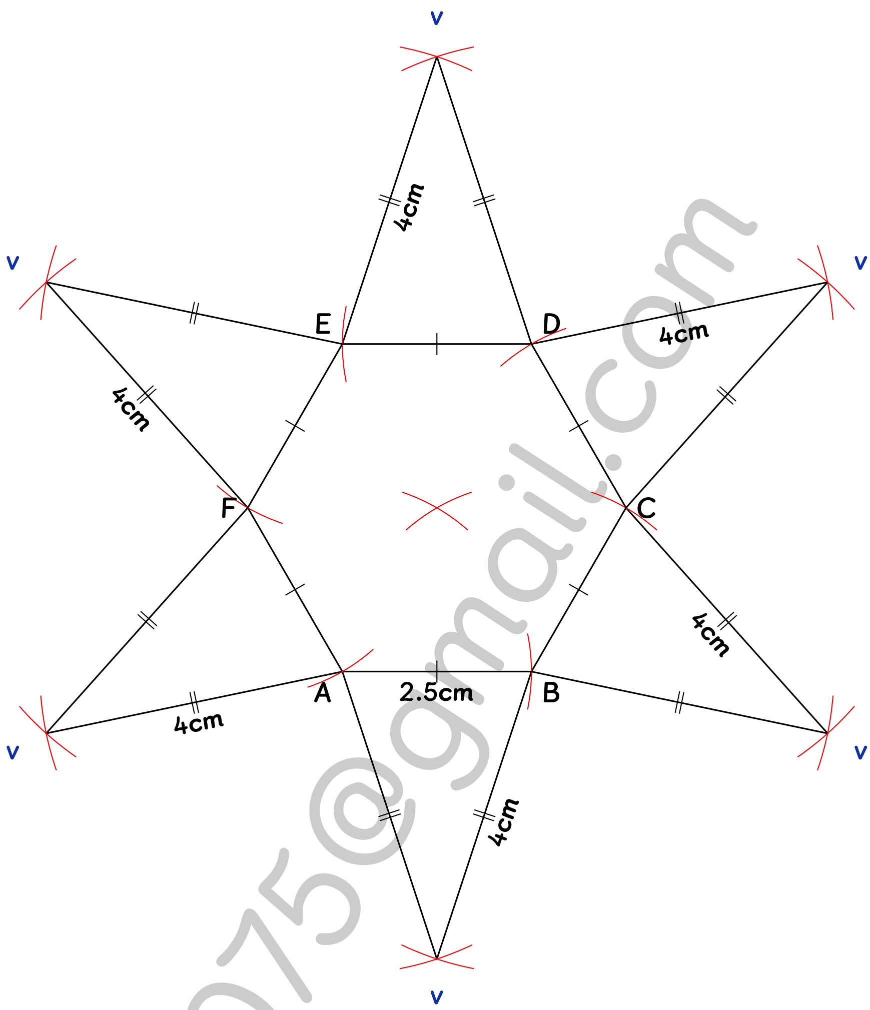
Draw a labelled net of the pyramid. (**3 marks**)

SOLUTION:

SOLID MARKING RUBRIC

Construction of hexagon $\checkmark B_1$

Construction of isosceles triangles $\checkmark B_1$



Problem 17 KCSE2022/P1/No. 17

A contractor hired Wema and Tatu to transport **144** tonnes of stones to building sites **A** and **B**.

To transport **48** tonnes of stones for a distance of **28km**, the contractor paid **Ksh. 24000**.

(a) Wema transported **96** tonnes of stones to site **A**, a distance of **49km**.

(i) Calculate the amount of money that was paid to Wema. (**2 marks**)

SOLUTION:

Let x = amount paid to Wema.

	Weight (tonnes)	Distance (km)	Pay (Ksh.)
Contractor	48	28	24000
	96	28	48000
Wema	96	49	x

$$28 \times x = 49 \times 48000 \quad \checkmark M_1$$

$$x = \frac{49 \times 48000}{28} = \textbf{Ksh. 84000} \quad \checkmark A_1$$

(ii) For every **8** tonnes of stones Wema transported to site **A**, he spent **Ksh. 3000**.
Calculate the profit Wema made. (**3 marks**)

SOLUTION:

$$\text{Expenses} = \frac{96}{8} \times 3000 = 36000 \checkmark M_1$$

$$\text{Profit} = 84000 - 36000 \quad \checkmark M_1$$

$$= \textbf{Ksh. } 48000 \quad \checkmark A_1$$

(b) Tatu transported the remaining **48** tonnes of stones to site **B**, a distance of **84km**. If Tatu made **44%** profit, calculate the amount of money Tatu spent to transport the stones. (**3 marks**)

SOLUTION:

Let y = amount paid to Tatu.

	Weight (tonnes)	Distance (km)	Pay (Ksh.)
Contractor	48	28	24000
Tatu	48	84	y

$$28 \times y = 84 \times 24000$$

$$y = \frac{84 \times 24000}{28} = \textbf{Ksh. } 72000 \checkmark M_1$$

$$\text{Expenses} = \frac{100 - 44}{100} \times 72000 \quad \checkmark M_1$$

$$= \textbf{Ksh. } 40320 \quad \checkmark A_1$$

(c) Determine the ratio of the profit made by Wema to that made by Tatu. (**2 marks**)

SOLUTION:

$$\text{Profit by Tatu} = 44\% \text{ of } 72000 = \textbf{Ksh. } 31680 \checkmark M_1$$

$$\text{Wema : Tatu} = 48000 : 31680$$

$$= 50 : 33 \quad \checkmark A_1$$

Problem 18 ■■■ KCSE2022/P1/No. 18

A shot put is spherical and has mass of **7.26 kg**. It is made of a metal with a density of **6.93g/cm^3**.

(Take $\pi = \frac{22}{7}$)

(a) Determine the radius of the shot put, correct to 1 decimal place. (**3 marks**)

SOLUTION:

Let r = radius of the shot put.

$$\text{volume} = \frac{7260}{6.93} = 1047.619$$

use a

$$\frac{4}{3} \times \frac{22}{7} \times r^3 = 1047.619 \quad \checkmark M_1$$

KEYPOINT

Volume of a sphere: $\frac{4}{3}\pi r^3$

$$r^3 = 1047.619 \times \frac{3}{4} \times \frac{7}{33} = 250 \checkmark M_1$$

$$r = \sqrt[3]{250} = 6.2996$$

use a

$$\approx 6.3\text{cm} \qquad \checkmark A_1$$

(b) A bucket is in the shape of a frustum of a cone. The base radius of the bucket is **7cm**. The bucket contains water to a height of **15cm**. The radius of the surface of the water is **10.5cm**.

(i) Find the volume of the water in the bucket. (**3 marks**)

SOLUTION:

Let **h** = height of the chopped off cone.

$$\frac{\text{Radius of bigger cone}}{\text{Radius of smaller cone}} = \frac{\text{Height of bigger cone}}{\text{Height of smaller cone}}$$

$$\frac{10.5}{7} = \frac{h+15}{h}$$

$$\implies h = 30 \implies h + 15 = 45\text{cm}$$

volume = vol of bigger cone − vol of smaller cone

$$= \frac{1}{3} \times \frac{22}{7}\left(10.5^2 \times 45 - 7^2 \times 30\right) \qquad \checkmark M_2$$

$$= 3657.5\text{cm}^3 \qquad \checkmark A_1$$

use c

KEYPOINT

Volume of a frustum:

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(R^2 H - r^2 h\right)$$

(ii) The shot put ball is completely submerged in the water in the bucket.

Calculate the new height of the water in the bucket. (**4 marks**)

SOLUTION:

Let **h** = height of new larger cone formed and **r** = new radius of water surface.

$$1047.619 = \frac{1}{3} \times \frac{22}{7}\left(r^2 h - 10.5^2 \times 45\right) \qquad \checkmark M_1$$

volu

$$\frac{\text{Height of new big cone}}{\text{Height of old smaller cone}} = \frac{\text{radius of new big cone}}{\text{radius of smaller cone}}$$

$$\frac{h}{45} = \frac{r}{10.5} \implies h = \frac{450}{105} r$$

$$1047.619 = \frac{1}{3} \times \frac{22}{7}\left\{r^2\left(\frac{450}{105} r\right) - 10.5^2 \times 45\right\} \checkmark M_1$$

$$1047.619 \times \frac{3}{1} \times \frac{7}{22} = \frac{450}{105} r^3 - 4961.25$$

$$\frac{450}{105} r^3 = 1047.619 + 4961.25 = 9452.5$$

$$r^3 = 9452.5 \times \frac{105}{450} = 1402.0694$$

$$r = \sqrt[3]{1402.0694} = 11.1924 \qquad \checkmark M_1$$

$$h = \frac{450}{105} \times 11.1924 = 47.9674$$

Therefore new height of water in the bucket is:

$$= 47.9674 - 30 = 17.9674\text{cm} \qquad \checkmark A_1$$

Problem 19 KCSE2022/P1/No. 19

A triangle **ABC** is right angled at point **A**. The vertices of the triangle are $A(1,-2)$, $B(5,4)$ and $C(m,n)$.

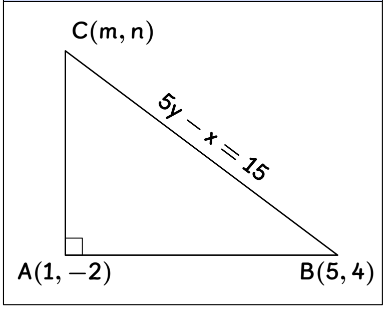
The equation of line **BC** is $5y - x = 15$.

(a) Determine:

(i) the equation of line **AC** in the form $ax+by+c=0$, where a, b and c are integers. **(4 marks)**

SOLUTION:

SKETCH



$$\text{gradient of AB} = \frac{4-(-2)}{5-1} = \frac{6}{4} = \frac{3}{2} \quad \checkmark M_1$$

$$\text{gradient of AC} = \frac{-1}{3/2} = -\frac{2}{3} \quad \checkmark M_1 \quad m_1 \times m_2 = -1$$

$$\text{equation of AC}: \quad y-(-2) = -\frac{2}{3}(x-1) \quad \checkmark M_1$$

$$\implies 2x+3y+4=0 \quad \checkmark A_1$$

(ii) the coordinates of point **C**. **(3 marks)**

SOLUTION:

$$5y - x = 15 \quad \times 2$$

$$-2x + 10y = 30 \quad \checkmark M_1 \quad \text{solve the two equations simultaneously}$$

$$2x + 3y = -4$$

$$13y = 26 \implies y = 2 \quad \checkmark M_1$$

$$x = 5(2) - 15 = -5$$

$$\therefore C(-5, 2) \quad \checkmark A_1$$

(b) A line passes through point **A** and is parallel to line **BC**.

Determine the **x**-intercept of the line. **(3 marks)**

SOLUTION:

$$\text{gradient of BC} = \frac{1}{5} \qquad y = \frac{1}{5}x + 3$$

$$\text{gradient of } \ell = \frac{1}{5} \qquad m_1 = m_2 \text{ if } \parallel$$

$$\text{equation of } \ell = y-(-2) = \frac{1}{5}(x-1) \quad \checkmark M_1$$

$$y = \frac{1}{5}x - \frac{11}{5}$$

$$x\text{-intercept}: \frac{1}{5}x = \frac{11}{5} \quad \checkmark M_1 \quad \text{at } x\text{-intercept } y = 0$$

$$x = 11$$

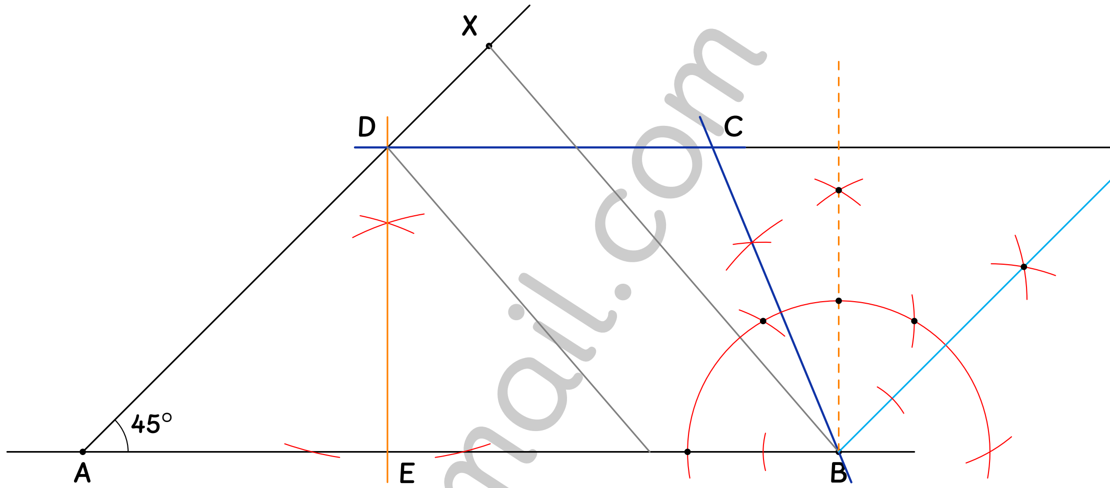
$$\implies x\text{-intercept} = 11 \quad \checkmark A_1$$

Problem 20 KCSE2022/P1/No. 20

In the figure below, line **AB** $=$ **10cm** and is part of a trapezium **ABCD**. Point X is such that angle **BAX** $=$ **45°**.

CONSTRUCTION MARKING RUBRIK

Join B to X	✓B_1
Locate point on AB at 7.5cm mark	
Draw parallel line though the point	✓B_1
Locate point D	✓B_1
Construct 135° at B	✓B_1
Bisect 135° at B to get 67.5°	✓B_1
Locate point C and draw line DC	✓B_1
Drop a ⊥ from D to AB	✓B_1



(a) Using a ruler and a pair of compasses only:

(i) locate point **D** on line **AX** such that **AD** : **DX** $=$ **3** : **1**. (**3 marks**)

(ii) complete trapezium **ABCD** such that line **DC** is parallel to line **AB** and angle **ABC** $=$ **67.5°**. (**3 marks**)

(iii) draw a perpendicular line from **D** to meet **AB** at **E**. Measure DE. (**2 marks**)

SOLUTION:

DE $=$**4.0cm** ✓B_1 ±0.1cm

(b) Calculate the area of the trapezium **ABCD**. (**2 marks**)

SOLUTION:

$$\text{Area of ABCD} = \frac{1}{2} \times DE(AB + CD)$$

$$= \frac{1}{2} \times 4(10 + 4) \quad ✓M_1 \quad DE = 4cm,\ AB = 10cm,\ CD = 4cm$$

$$= 28cm^2 \quad ✓A_1$$

Problem 21 KCSE2022/P1/No. 21

The amount of money, in Kenya shillings, spent on airtime by a group of 30 people in a period of an hour was recorded ass shown below.

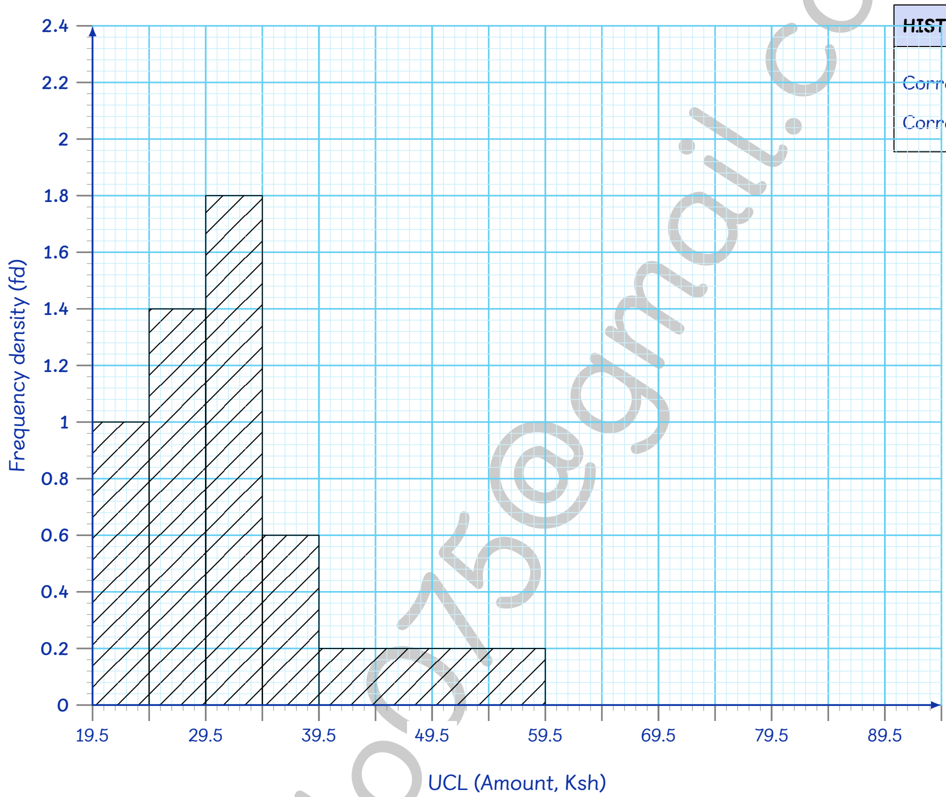
27	20	21	24	22	25
42	34	55	26	30	39
35	46	32	21	38	34
31	37	27	29	32	56
33	44	25	31	28	30

(a) Complete the frequency distribution table below. (**2 marks**)

Tally	𝍸	𝍸 II	𝍸 IIII	𝍸 I	III
Amount(**Ksh.**)	20 – 24	25 – 29	30 – 34	35 – 44	45 – 59
Frequency	5	7	9	6	3
Frequency Density	1	1.4	1.8	0.6	0.2

✓B_2

(b) On the grid below, draw a histogram to represent the data. (**3 marks**)



HISTOGRAM MARKING RUBRIC	
Correct scale used	✓S_1
Correct bars drawn	✓B_2

(c) Use the histogram to determine:

(i) The median amount of money spent on airtime by the **30** people.(**3 marks**)

SOLUTION:

$$5 \times 1.0 + 5 \times 1.4 + 1.8x = \frac{1}{2} \times 30 = 15 \qquad ✓M_1$$

$$12 + 1.8x = 15$$

$$1.8x = 3 \implies x = 1\frac{2}{3} = 1.667 \qquad ✓M_1$$

$$\text{median} = 29.5 + 1.667 = 31.167$$

$$= 31.17 \qquad ✓A_1$$

(ii) the number of people who spent more than **Ksh. 41.50** on airtime over that period. (**2 marks**)

SOLUTION:

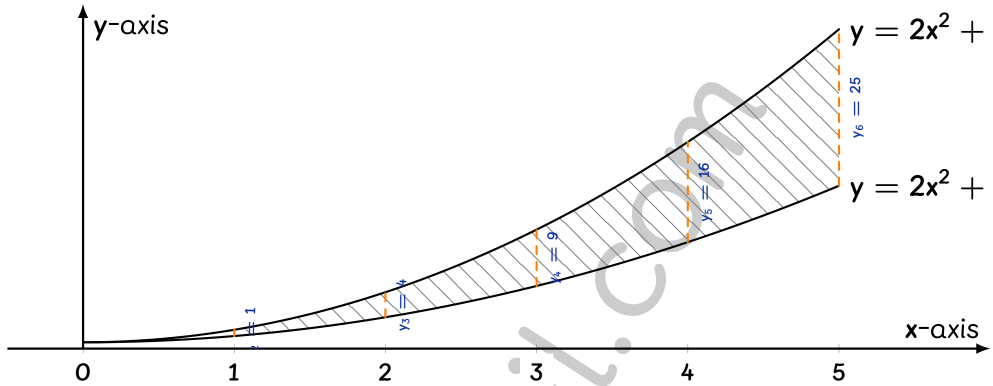
$$\text{number over } 41.5 = 0.6 \times 5 + 0.2 \times 15 \quad ✓M_1$$

$$= 6 \text{ people} \qquad ✓A_1$$

Problem 22

KCSE2022/P1/No. 22

The diagram below is a sketch of two curves $y = 2x^2 + 1$ and $y = x^2 + 1$ drawn on the same grid.



(a) Using the trapezium rule with 5 strips, estimate the area bounded by the curves $y = 2x^2 + 1$, $y = x^2 + 1$ and the lines $x = 0$ and $x = 5$. (**5 marks**)

SOLUTION:

x	0	1	2	3	4	5	
$h_1 = 2x^2 + 1$	1	3	9	19	33	51	
$h_2 = x^2 + 1$	1	2	5	10	17	26	✓M_1
$y_i = (h_1 - h_2)$	0	1	4	9	16	25	✓M_1

$$\text{Area} = \frac{1}{2}h\left\{(y_1 + y_6) + 2(y_2 + y_3 + y_4 + y_5)\right\}$$

$$= \frac{1}{2}\left\{(0 + 25) + 2(1 + 4 + 9 + 16)\right\} \quad ✓M_2$$

$$= 42\frac{1}{2} \text{ square units} \quad ✓A_1$$

use a

(b) Using the mid ordinate rule with 5 strips, estimate the area bounded by the curves $y = 2x^2 + 1$, $y = x^2 + 1$ and the lines $x = 0$ and $x = 5$. (**5 marks**)

SOLUTION:

x	0.5	1.5	2.5	3.5	4.5	
$h_1 = 2x^2 + 1$	1.5	5.5	13.5	25.5	41.5	
$h_2 = x^2 + 1$	1.25	3.25	7.25	13.25	21.25	✓M_1
$y_i = (h_1 - h_2)$	0.25	2.25	6.25	12.25	20.25	✓M_1

$$\text{Area} = h\,(y_1 + y_2 + \cdots + y_5)$$

$$= 1(0.25 + 2.25 + 6.25 + 12.25 + 20.25) \quad ✓M_2$$

$$= 41\frac{1}{4} \text{ square units} \quad ✓A_1$$

use a

Problem 23

KCSE2022/P1/No. 23

A supermarket sold **530** packets of milk daily when the price was **Ksh. 50** per packet.

Whenever the price per packet increased by **Ksh. 4**, the number of packets sold decreased by **20**.

If **n** represents the number of times the price was increased:

(a) Write an expression in terms of **n** for:

(i) the price of a packet of milk after the price was increased. (**1 mark**)

SOLUTION:

$$=50+4n \checkmark B_1$$

(ii) the number of packets of milk sold after the price was increased. (**1 mark**)

SOLUTION:

$$=530-20n \checkmark B_1$$

(iii) the total sales, in simplified expanded form, after the price of a packet of milk was increased. (**2 marks**)

SOLUTION:

$$S=(50+4n)(530-20n) \checkmark M_1 \quad \text{let } S=\text{total sales}$$

$$=-80n^2+1120n+26500 \checkmark A_1$$

(b) Determine

(i) the number of times the price was increased to attain maximum sales. (**3 marks**)

SOLUTION:

$$\frac{dS}{dn}=-160n+1120 \checkmark M_1$$

$$0=-160n+1120 \checkmark M_1 \quad \text{at maximum sales } \frac{dS}{dn}=0$$

$$\implies n=7 \checkmark A_1$$

(ii) the price of a packet of milk from maximum sales. (**1 mark**)

SOLUTION:

$$50+4\times 7=\text{Ksh. }78 \checkmark B_1$$

(iii) the maximum sales. (**2 marks**)

SOLUTION:

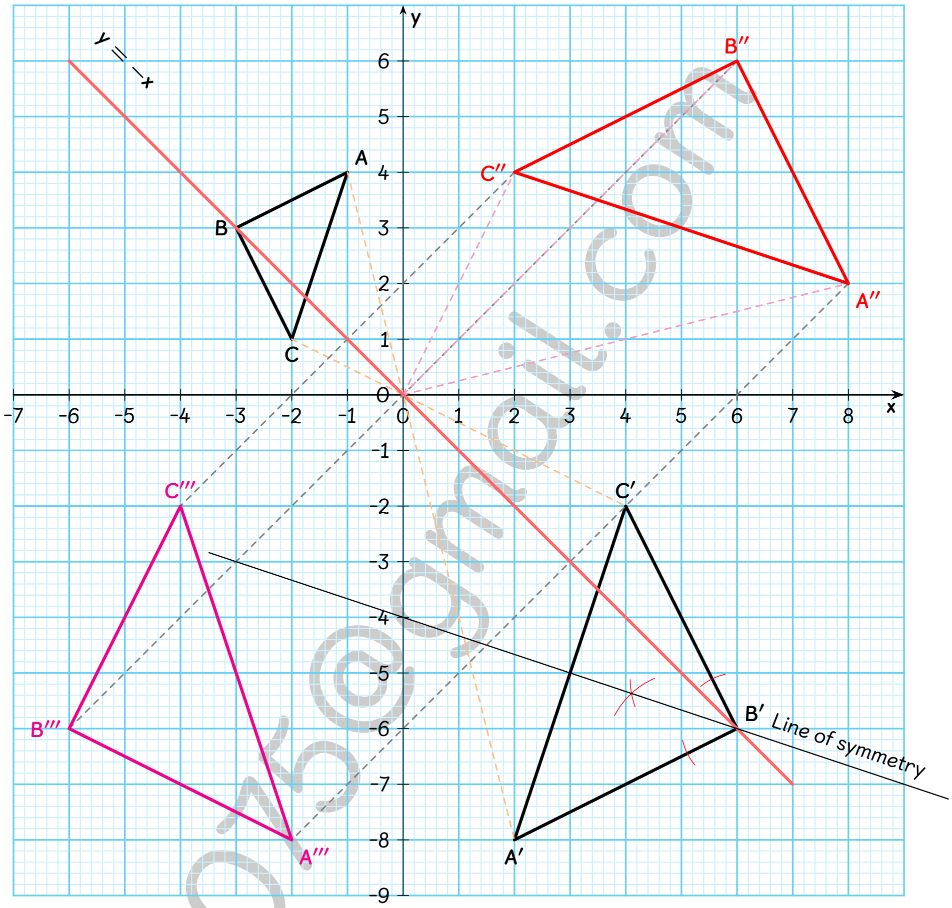
$$S=-80(7)^2+1120(7)+26500 \checkmark M_1 \quad \text{substitute } n=7 \text{ into } S$$

$$=\text{Ksh. }30420 \checkmark A_1$$

Problem 24 KCSE2022/P1/No. 24

Triangle **ABC** and **A′B′C′** are drawn on the grid provided.

MARKING RUBRIC	
Triangle A″B″C″ drawn construction marks must be seen	✓B_2
Triangle A‴B‴C‴ drawn construction marks must be seen	✓B_2
Bisect ∠A′B′C′	✓B_1



(a) Describe fully a single transformation that mapped triangle **ABC** onto triangle **A′B′C′**. (**2 marks**)

SOLUTION:

Enlargement, scale factor -2 and centre $(0, 0)$ ✓B_2

evidence must be shown on the grid

(b) On the same grid, draw:

(i) triangle **A″B″C″** the image of **A′B′C′** under a rotation of $+90^\circ$ about $O(0, 0)$. (**2 marks**)

SOLUTION:

$A''(8, 2)$, $B''(6, 6)$, $C''(2, 4)$

(ii) triangle **A‴B‴C‴**, the image of triangle **A″B″C″** under a reflection in the line $y = -x$ (**2 marks**)

SOLUTION:

$A'''(-2, -8)$, $B'''(-6, -6)$, $C'''(-4, -2)$

(c) Draw the line of symmetry of triangle **A′B′C′** and hence determine its equation in the form $y = mx + c$, where **m** and **c** are constants. (**4 marks**)

SOLUTION:

$(6, -6)$ and $(3, -5)$

$$m = \frac{-5-(-6)}{3-6} = -\frac{1}{3} \quad \checkmark M_1$$

equation of symmetry: $y-(-5) = -\frac{1}{3}(x-3)$ ✓M_1

$$\implies y = -\frac{1}{3}x - 4 \quad \checkmark A_1$$

KCSE 2022 Paper 2

Problem 25 ■■■ KCSE2022/P2/No. 1

An investor took a loan from a bank that charged interest. the loan and the interest accrued were repaid in monthly instalments. The investor repaid **Ksh. 1500** in the first month and in each subsequent month the instalments were reducing by **Ksh. 50** until the loan was fully repaid. Determine the maximum amount that may be paid for that loan. (**3 marks**)

SOLUTION:

Let n be the number of months it takes to reduce the monthly instalments to 0.

This problem reduces to computing the number of term of the AP.

$a = 1500, d = -50, T_n = 0$

$T_n = a + (n-1)d$ explicit AP nth term formula

$0 = 1500 + (n-1)(-50)$

$\implies n = 31$ ✓M_1

Let S_n be the total amount of money paid for the loan.
This problem reduces to computing the sum of the AP

$S_{30} = \frac{n}{2}(2a + (n-1)d)$ explicit AP sum formula

$= \frac{31}{2}(2 \times 1500 + (31-1)(-50))$ ✓M_1 substitute known values $n = 31, a = 1500, d = -50$

$=$**Ksh. 23250** ✓A_1 use a

Problem 26 ■■■ KCSE2022/P2/No. 2

Two machines A and B working independently can take 8 hours and 10 hours respectively to do a task. A third machine C and machine A working together can do the same task in 5 hours. Determine the time it would take machine B and machine C working together to do the same task. (**3 marks**)

SOLUTION:

Let t = time it takes machine C to do a task alone.

	Total time in hours	Fractional part done in 1 hours	Fractional part each machine does in 5 hours
Machine A	8	$\frac{1}{8}$	$\frac{5}{8}$
Machine B	10	$\frac{1}{10}$	—
Machine C	t	$\frac{1}{t}$	$\frac{5}{t}$

Since it takes both in machines 5 hours to complete the task.

$$\frac{5}{8}+\frac{5}{t}=1$$

$$\frac{5}{t}=\frac{3}{8}$$

$$\implies t=\frac{8}{3}\times\frac{5}{1}=\frac{40}{3} \quad \checkmark M_1$$

Let m = time it takes both B and C working together.

$$\frac{m}{10}+\frac{5m}{t}=1$$

$$m\left(\frac{1}{19}+\frac{5}{t}\right)=1$$

$$m=1\div\left(\frac{1}{19}+\frac{5}{40/3}\right) \quad \checkmark M_1 \quad \text{substitute } t=\frac{4}{45}$$

$$m=\frac{1}{7/40}=\frac{40}{7}$$

$$=5\frac{5}{7} \quad \checkmark A_1$$

Both machine B and C would take $5\frac{5}{7}$ hours to do the task together.

Problem 27 ■ ■ ■ KCSE2022/P2/No. 3

Simplify $\frac{3+\sqrt{5}}{7-3\sqrt{5}}$, leaving the answer in the form $a+b\sqrt{c}$ where a, b and c are integers. (**2 marks**)

SOLUTION:

$$\frac{3+\sqrt{5}}{7-3\sqrt{5}}=\frac{3+\sqrt{5}}{7-3\sqrt{5}}\times\frac{7+3\sqrt{5}}{7+3\sqrt{5}} \quad \checkmark M_1 \quad \text{multiply by the rational conjugate}$$

$$=\frac{21+9\sqrt{5}+7\sqrt{5}+15}{49-45}$$

$$=\frac{36+16\sqrt{5}}{4}$$

$$=9+4\sqrt{5} \quad \checkmark A_1$$

Problem 28 ■ ■ ■ KCSE2022/P2/No. 4

The market value of a certain precious stone varies directly as the square of its mass. One such stone of mass **10 kg** has a value of **Ksh. 600000**.

Calculate the value of a similar stone whose mass is **18.5 kg**. (**3 marks**)

SOLUTION:

Let P = market value and **m** = mass of stone

$$P \propto m^2 \implies P = km^2$$ k = constant of proportionality

$$600000 = k(10)^2$$ substitute P = 600000 and **m** = 10

$$\implies k = 6000$$ solve for k

$$\therefore P = 6000m^2$$ ✓M_1 substitute k = 6000 to get the defining equation

Hence at **m** = **18.5 kg**;

$$P = 6000(18.5)^2$$ ✓M_1 substitute **m** = 18.5

$$= \textbf{Ksh. 2053500}$$ ✓A_1 use a

Problem 29 KCSE2022/P2/No. 5

The perimeter of a rectangle is **48cm** while its area is **108cm²**. Form a quadratic equation to represent the situation and hence determine the dimensions of the rectangle. (**3 marks**)

SOLUTION:

Let **a** = length of rectangle and **b** = width of rectangle.

$$2(a+b) = 48$$ from perimeter = **48cm**

$$ab = 108$$ from area = **108cm²**

$$a(24-a) = 108$$ ✓M_1 substitute **b** = 24 − a

$$24a - a^2 = 108$$

$$a^2 - 24a + 108 = 0$$

$$(a-6)(a-18) = 0$$ ✓M_1

$$a - 6 = 0 \implies a = 6$$

$$a - 18 = 0 \implies a = 18$$

When $a = 6$, $y = 24 - 6 = 18$

When $a = 18$, $y = 24 - 18 = 6$

Hence the dimensions are **6cm** by **18cm**. ✓A_1

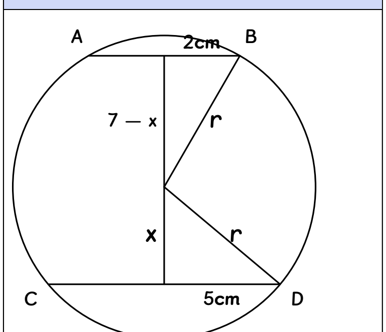
Problem 30 KCSE2022/P2/No. 6

Two parallel chords AB = **4cm** and CD = **10cm** lie on opposite sides of a centre **O** of a circle. The perpendicular distance between the two chords is **7cm**.

Calculate the radius of the circle leaving the answer in surd form. (**3 marks**)

SOLUTION:

SKETCH



Let **h** = distance from centre **O** to midpoint of **CD**.

$\therefore$ 7 − **h** = distance from centre **O** to midpoint of **AB**.

$$OB^2 = (7-h)^2 + 2^2$$

$$= 49 - 14h + h^2 + 4$$ ✓M_1

$$OD^2 = h^2 + 5^2$$

$$\implies 53 - 14h + h^2 = h^2 + 25 \quad \checkmark M_1$$

$$14h = 28 \implies h = 2$$

$$\therefore OD = \sqrt{2^2 + 5^2}$$

$$= \sqrt{29} \quad \checkmark A_1$$

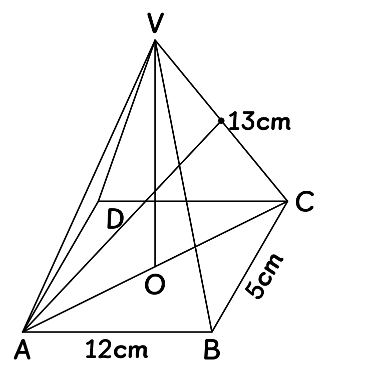
Problem 31 KCSE2022/P2/No. 7

A rectangle **ABCD** in which **AB** = **12cm** and **BC** = **5cm** is the base of a right pyramid whose apex is **V**. **VA** = **VB** = **VC** = **VD** = **13cm**. Point **M** is the mid point of the edge **VC**.

Calculate, correct to 2 decimal places, the length of line **AM**. (**3 marks**)

SOLUTION:

SKETCH



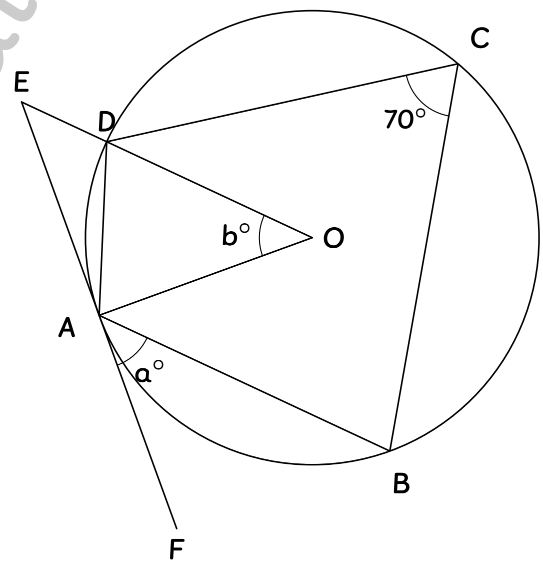
$$AC = \sqrt{12^2 + 5^2} = 13 \quad \checkmark M_1$$ so $\triangle AVC$ is equilateral since $VA = VC = AC$

$$AM = \sqrt{13^2 - 6.5^2} \quad \checkmark M_1$$

$$= 11.26cm \quad \checkmark A_1$$

Problem 32 KCSE2022/P2/No. 8

In the figure below, **O** is the centre of the circle. Points **A**, **B**, **C** and **D** lie on the circumference of the circle. Line **AB** is parallel to the straight line **EDO** and line **FAE** is a tangent to the circle at **A**. $\angle FAB = a^\circ$, $\angle DOA = b^\circ$, $\angle DCB = 70^\circ$.



Determine the values of **a** and **b**. (**4 marks**)

SOLUTION:

$$\angle DAB = 180^\circ - 70^\circ = 110^\circ$$ opposite angles are supplementary

$$\angle DAO = \frac{180^\circ - b}{2}$$ base angles of isosceles triangle are equal

$$\angle OAB = \angle DOA = b$$ alternating angles are equal

$$110^\circ = \frac{180^\circ - b}{2} + b$$ ✓M_1 $\angle DAB = \angle DAO + \angle OAB$

$$\implies b = 40$$ ✓M_1

$$\angle OAF = \angle FAB + \angle BAO = a + b$$

$$90^\circ = a + 40^\circ$$ ✓M_1 $\angle OAF = 90^\circ$ since $EF \perp OA$

$$\implies a = 50$$ ✓A_1

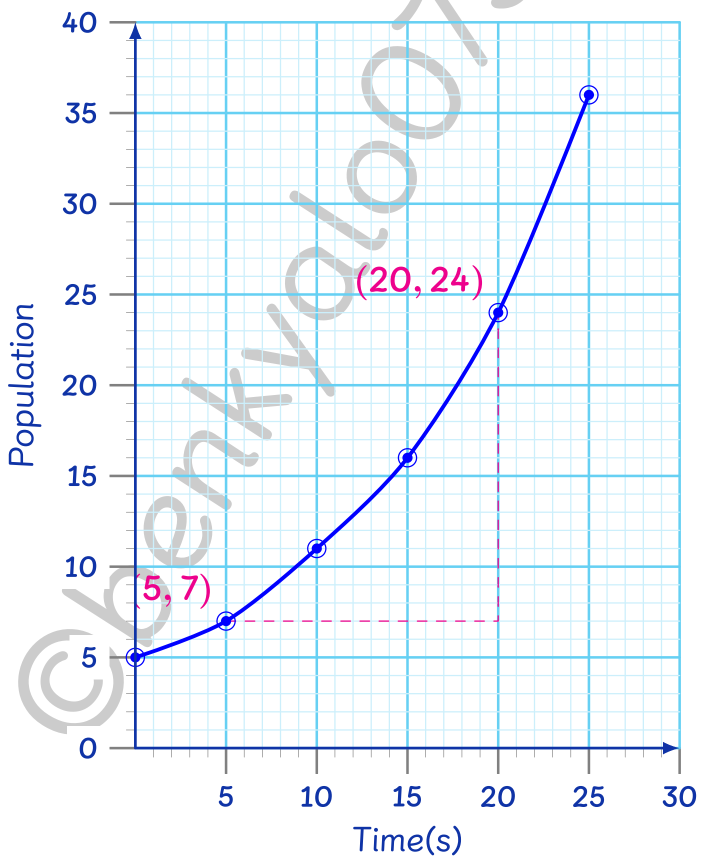
Problem 33

KCSE2022/P2/No. 9

The population growth of a colony of bacteria was recorded at intervals of 5 seconds as shown in the table below.

t(s)	0	5	10	15	20	25
Number of bacteria	5	7	11	16	24	36

(a) On the grid provided, draw a graph of the population of bacteria against time. (**2 marks**)



GRAPH MARKING RUBRIC	
All points plotted	✓P_1
Line of best fit drawn	✓L_1

(b) Use the graph to determine, correct to 2 decimal places, the average rate of change of the population of bacteria between $t = 5$ seconds and $t = 20$ seconds.. (**2 marks**)

SOLUTION:

Using $(20, 24)$ and $(5, 7)$

$$\text{gradient} = \frac{24-7}{20-5} \quad \checkmark M_1$$

$$= \frac{17}{15} = 1.13 \quad \checkmark A_1 \quad \text{use a}$$

Problem 34 KCSE2022/P2/No. 10

A circle centre $C(5, 5)$ passes thought points $A(1, 3)$ and $B(a, 9)$. Find the equation of the circle and hence the possible values of a. **(3 marks)**

SOLUTION:

$$r^2 = (5-1)^2 + (5-3)^2 = 20 \qquad r = \text{radius of circle}$$

$$\text{Equation of circle CA: } (x-5)^2 + (y-5)^2 = 20 \quad \checkmark M_1 \qquad (x-a)^2 - (y-b)^2 = r^2$$

$$(a-5)^2 + (9-5)^2 = 20 \qquad \text{substitute } x = a \text{ and } y = 9$$

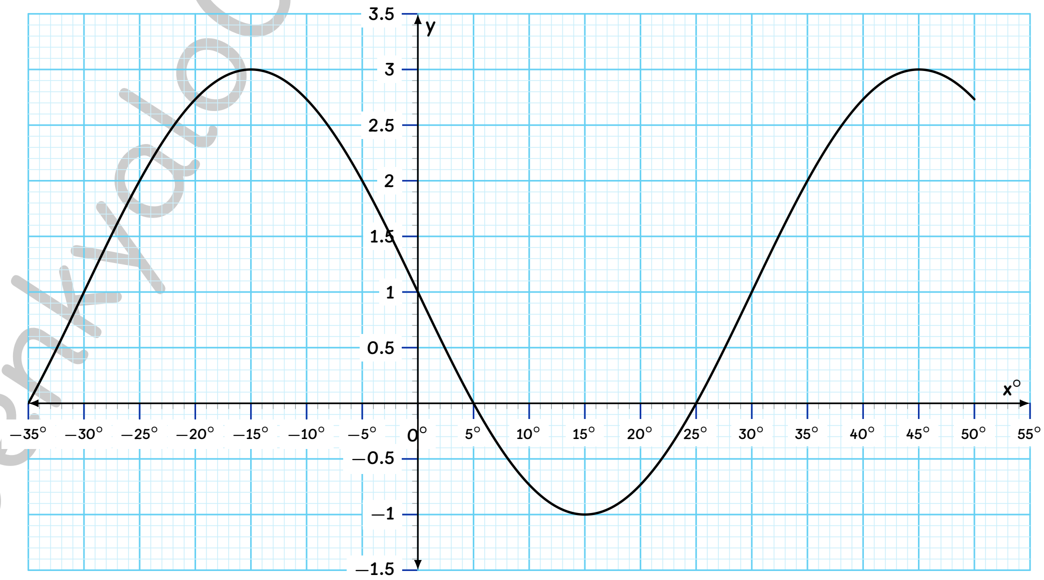
$$(a-5)^2 = 20 - 16 = 4 \quad \checkmark M_1$$

$$a - 5 = \pm\sqrt{4} = \pm 2$$

$$a = 7 \text{ or } 3 \quad \checkmark A_1$$

Problem 35 KCSE2022/P2/No. 11

The figure below represents the curve of the function $y = 1 - A\sin wx$ for the range $-35° \le x \le 50°$.



Determine the values of A and R. **(3 marks)**

SOLUTION:

$$A = \frac{3-(-1)}{2} = 2 \quad \checkmark B_1 \quad \text{amplitute}$$

$$\text{period} = 25 - (-35) = 60° \qquad \text{observe from the graph}$$

SKILL HUNT

For any sine function in the form $y = a\sin(bx + c)$, then:

- Amplitude is a,
- period is $\frac{2\pi}{b} = \frac{360}{b}$,
- and phase is given by c.

$$60 = \frac{360}{w}$$

$$\implies w = 6 \quad \checkmark B_1$$

Problem 36 KCSE2022/P2/No. 12

The data below represents the number of animals owned by 7 neighbours:

9, 5, 14, 6, 8, 13 and 15.

Calculate, correct to the nearest whole number, the standard deviation of the number of animals. (**3 marks**)

SOLUTION:

$$\bar{x} = \frac{\sum x}{n} = \frac{9+5+14+6+8+13+15}{7} = 10$$ use a

$$d: -1,\ 5,\ 4,\ -4,\ -2,\ 3,\ 5 \quad \checkmark M_1$$ $d = x - \bar{x}$

$$d^2: 1,\ 25,\ 16,\ 16,\ 4,\ 9,\ 25$$

$$s = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{1+25+16+16+4+9+25}{7}} \quad \checkmark M_1$$

$$= \sqrt{\frac{96}{7}} = 3.7034$$ use a

$$\approx 4 \quad \checkmark A_1$$ nearest whole number

Problem 37 KCSE2022/P2/No. 13

The table below shows income tax rates in a certain year.

Monthly taxable income in Kenya shillings	Tax rates in each shilling (%)
0 − 12298	10
12299 − 23885	15
23886 − 35472	20

A tax relief of **Ksh. 1408** per month was allowed. Calculate the monthly income tax paid by an employee whose monthly taxable income was **Ksh. 26545.75**. (**3 marks**)

SOLUTION:

1st band: $10\% \times 12298 = $ **Ksh. 1229.80**

2nd band: $15\% \times 11587 = $ **Ksh. 1738.05** $\checkmark M_1$

3rd band: $20\% \times 2660.75 = $ **Ksh. 532.15**

Gross tax $= 1229.80 + 1738.05 + 532.15$ $\checkmark M_1$

$=$ **Ksh. 3500**

Net tax $= 3500 - 1408$

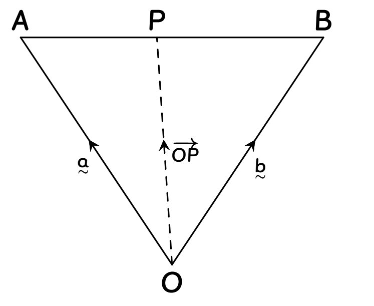
$=$ **Ksh. 2092** $\checkmark A_1$

Problem 38 KCSE2022/P2/No. 14

Point $P(8, 4, -1)$ divides line AB internally in the ration $4 : 1$. The position vector of point A with respect to the origin O is $\begin{pmatrix} -4 \\ 8 \\ 3 \end{pmatrix}$. Determine the coordinates of point B. (**3 marks**)

SOLUTION:

SKETCH



$$\overrightarrow{OP} = \frac{4}{5}\overrightarrow{OB} + \frac{1}{5}\overrightarrow{OA}$$

$$\begin{pmatrix} 8 \\ 4 \\ -1 \end{pmatrix} = \frac{4}{5}\overrightarrow{OB} + \frac{1}{5}\begin{pmatrix} -4 \\ 8 \\ 3 \end{pmatrix} \quad \checkmark M_1$$

$$\frac{4}{5}\overrightarrow{OB} = \begin{pmatrix} 8 \\ 4 \\ -1 \end{pmatrix} - \begin{pmatrix} -4/5 \\ 8/5 \\ 3/5 \end{pmatrix} = \begin{pmatrix} 44/5 \\ 12/5 \\ -8/5 \end{pmatrix}$$

$$\overrightarrow{OB} = \frac{5}{4}\begin{pmatrix} 44/5 \\ 12/5 \\ -8/5 \end{pmatrix} = \begin{pmatrix} 11 \\ 3 \\ -2 \end{pmatrix} \quad \checkmark A_1$$

Hence coordinate of B is $(11, 3, -2)$ $\checkmark B_1$

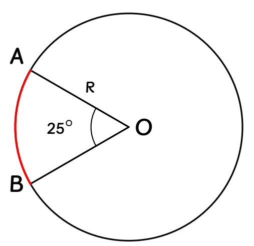
Problem 39 KCSE2022/P2/No. 15

An aircraft took off from an airport $A(0^\circ, 40^\circ W)$ at **1100h** local time. The aircraft landed at airport $B(0^\circ, 65^\circ W)$ at **1200h** local time.

Determine the speed of the aircraft in knots. (**4 marks**)

SOLUTION:

KEYPOINT



longitude difference $= 65 - 40 = 25^\circ$ $\checkmark B_1$ same side hence subtract

time difference $= 25 \times 4 = 100\text{ min} = 1h40\text{ min}$

arc length $AB = 60 \times 25 = 1500nm$ $\checkmark M_1$ Distance $= 60\theta$

time taken $= 1200h - 1100h + 1h40\text{ min}$

$$= 2h40\text{ min} = 2\frac{2}{3}h = \frac{8}{3}h$$

speed $= 1500nm \div \frac{8}{3}$ $\checkmark M_1$ speed $= \dfrac{\text{distance}}{\text{time taken}}$

$= 562.5$ knots $\checkmark A_1$

Problem 40 KCSE2022/P2/No. 16

The velocity v **m/s** of a particle moving in a straight line is $(-2t+4)$ **m/s**. Determine the distance moved by the particle during the first second of its motion. (**3 marks**)

SOLUTION:

$$s = \int vdt = \int_0^1 (-2t+4)dt \checkmark B_1$$ during first second is between $t = 0$ and $t = 1$

$$= \left[-t^2 + 4t \right]_0^1 \checkmark M_1$$

$$= -1 + 4 = 3\,m \checkmark A_1$$

Problem 41 KCSE2022/P2/No. 17

A wholesaler stocks two types of rice: Refu and Tamu. the wholesale prices of **1 kg** of Refu and **1 kg** of Tamu are **Ksh. 80** and **Ksh. 140** respectively. The wholesaler also stocks blend **A** rice which is a mixture of Refu and Tamu rice mixed in the ration **3 : 2**.

(a) (i) A retailer bought **10 kg** of blend **A** rice. To this blend, the retailer added some Tamu rice to prepare a new mixture blend **X**. the ratio of Refu rice to Tamu rice in blend **X** was **1 : 2**.
Determine the amount of Tamu rice that was added. (**3 marks**)

SOLUTION:

Let a = amount of Tamu rice added to prepare blend **X**.

	Refu	Tamu	Mixture
Ratio (as a fraction)	$\frac{3}{5}$	$\frac{2}{5}$	1
Quantity(**kg**)	$\frac{3}{5} \times 10$	$\frac{2}{5} \times 10$	10
Blend X	$\frac{3}{5} \times 10$	$\frac{2}{5} \times 10 + a$	$10 + a$

The ratio of Refu rice to Tamu rice in blend **X** is **1 : 2**, hence

$$\frac{\frac{3}{5} \times 10}{\frac{2}{5} \times 10 + a} = \frac{1}{2} \quad \checkmark M_1$$

$$\frac{6}{4+a} = \frac{1}{2} \quad \checkmark M_1$$

$$6 \times 2 = 4 + a$$

$$\Longrightarrow \quad a = 12 - 4 = 8\,\text{kg} \checkmark A_1$$

(ii) The retailer sold blend **X** rice making a profit of **20%**. Determine the selling price of **1 kg** of blend **X**. (**3 marks**)

SOLUTION:

$$\text{Buying Price per } \textbf{kg} = \frac{6 \times 80 + (8+4) \times 140}{18} \checkmark M_1$$

$$= \textbf{Ksh. 120}$$

$$\text{Selling price per } \textbf{kg} = 120\% \times 120 \quad \checkmark M_1$$

$$= \textbf{Ksh. 144} \quad \checkmark A_1$$

(b) The wholesaler prepared another mixture, blend **B**, by mixing **x kg** of blend **A** rice with **y kg** of Tamu rice. Blend **B** has a wholesale price of **Ksh. 130** per kg.
Determine the ratio **x : y**. (**4 marks**)

SOLUTION:

	Blend A	Tamu rice	Mixture
Ratio (as a fraction)	$\frac{x}{x+y}$	$\frac{y}{x+y}$	1
Buying price	$\frac{80 \times 3 + 140 \times 2}{5}$	144	–
Total Cost	$\frac{x}{x+y} \times 104$	$\frac{y}{x+y} \times 144$	130

Selling price per **kg** was **Ksh.** 130, hence

$$\frac{x}{x+y} \times 104 + \frac{y}{x+y} \times 140 = 130 \quad \checkmark M_1$$

$$\frac{104x + 140y}{x+y} = 130$$

$$104x + 140y = 130x + 130y \checkmark M_1$$

$$26x = 10y$$

$$\frac{x}{y} = \frac{10}{26} = \frac{5}{13} \quad \checkmark M_1$$

Hence the ratio **x : y** is **5 : 13** $\checkmark A_1$

Problem 42

KCSE2022/P2/No. 18

Two bags **P** and **Q** contain identical marbles except for the colours. Bag **P** contains **3** green and **4** read marbles. Bag **Q** contains **2** green and **3** red marbles.

(a) Find the probability of picking a red marble from bag **P**. (**1 mark**)

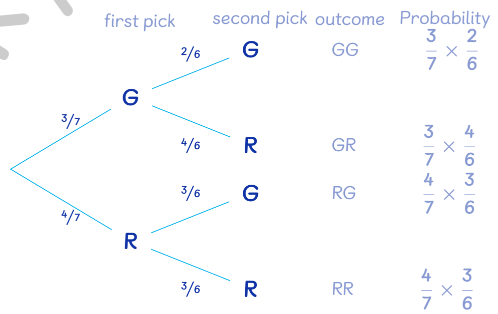
SOLUTION:

$$P(\text{red ball}) = \frac{4}{7} \checkmark B_1$$

(b) Two marbles were picked at random from bag **P**, one at a time, without replacement.

(i) Draw a probability tree diagram to show all the possible outcomes. (**1 mark**)

SOLUTION:



G : Red ball $\checkmark B_1$
R : Blue ball

(ii) Find the probability that the two marbles picked were of the same colour. (**2 marks**)

SOLUTION:

$$P(GG) \text{ or } P(RR) = P(GG) + P(RR) \qquad \text{P(same colour)}$$

$$= \frac{3}{7} \times \frac{2}{6} + \frac{4}{7} \times \frac{3}{6} \checkmark M_1$$

$$= \frac{3}{7} \quad ✓A_1$$

(iii) Find the probability that at least one red marble was picked. (**2 marks**)

SOLUTION:

P(at least one red marble)

$$P(GR) \text{ or } P(RG) \text{ or } P(RR) = 1 - P(GG)$$

$$= 1 - \frac{3}{7} \times \frac{2}{6} \quad ✓M_1$$

$$= \frac{6}{7} \quad ✓A_1$$

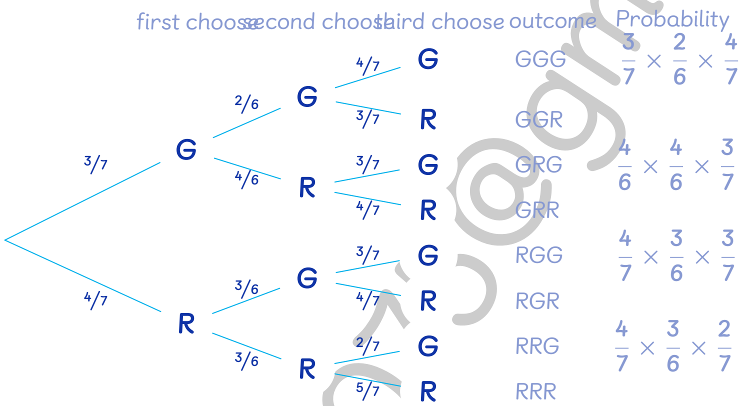
use a

(c) The marbles picked from bag **P** in (b) were both put into bag **Q**. A marble was then picked at random from bag **Q**.

Calculate the probability that the marble picked was:

(i) green in colour. (**3 marks**)

SOLUTION:



G : Green ball

R : Red ball

$$P(\text{green}) = P(GGG) + P(GRG) + P(RGG) + P(RRG)$$

$$= \frac{3}{7} \times \frac{2}{6} \times \frac{4}{7} + \frac{4}{6} \times \frac{4}{6} \times \frac{3}{7} + \frac{4}{7} \times \frac{3}{6} \times \frac{3}{7} + \frac{4}{7} \times \frac{3}{6} \times \frac{2}{7} \quad ✓M_2$$

$$= \frac{20}{49} \quad ✓A_1$$

(ii) red in colour. (**3 marks**)

SOLUTION:

$$P(\text{red}) = 1 - \frac{20}{49} \quad ✓M_2$$

$$= \frac{29}{49} \quad ✓A_1$$

Problem 43

KCSE2022/P2/No. 19

A transformation matrix $T_1 = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$ maps a triangle **ABC** onto triangle **A′B′C′**. Another transformation matrix $T_2 = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ maps triangle **A′B′C′** into triangle **A″B″C″**. The coordinates of point **C″** is **(10, 8)** and the area of triangle **A″B″C″** is **15** square units.

(a) (i) Determine the coordinates of **C**. (**5 marks**)

SOLUTION:

$$T = T_1 \quad T_2$$

$$= \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 4.5 & -4 \\ 3 & -2 \end{pmatrix} \checkmark M_1$$

$$TC = C'' \implies C = T^{-1}C''$$

$$T^{-1} = \frac{1}{3}\begin{pmatrix} -2 & 4 \\ -3 & 4.5 \end{pmatrix} = \begin{pmatrix} -2/3 & 4/3 \\ -1 & 3/2 \end{pmatrix} \checkmark M_1$$

$$C = T^{-1} \quad C''$$

$$= \begin{pmatrix} -2/3 & 4/3 \\ -1 & 3/2 \end{pmatrix}\begin{pmatrix} 10 \\ 4 \end{pmatrix} \checkmark M_1$$

$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} \checkmark A_1$$

Hence the coordinates of C are $(4, 2)$ $\checkmark B_1$

(ii) Determine the area of triangle ABC. (**3 marks**)

SOLUTION:

$$ASF = \det\begin{pmatrix} 4.5 & -4 \\ 3 & -2 \end{pmatrix} = 3 \checkmark M_1$$

$$\frac{\text{Area of } A''B''C''}{\text{Area of } ABC} = 3$$

$$\frac{15}{\text{Area of } ABC} = 3 \checkmark M_1$$

$$\text{Area of } ABC = \frac{15}{3} = 5 \text{ square units} \checkmark A_1$$

(b) The coordinates of points B and B″ are (x, y) and $(6x+1, 8)$ respectively. Determine the value of y. (**2 marks**)

SOLUTION:

$$T \quad B = B''$$

$$\begin{pmatrix} 4.5 & -4 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6x+1 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} 4.5x - 4y \\ 3x - 2y \end{pmatrix} = \begin{pmatrix} 6x+1 \\ 8 \end{pmatrix} \checkmark M_1$$

$$\implies 4.5x - 4y = 6x + 1 \implies 1.5x + 4y = -1 \qquad \times 2$$

$$3x + 8y = -2$$

$$3x - 2y = 8$$

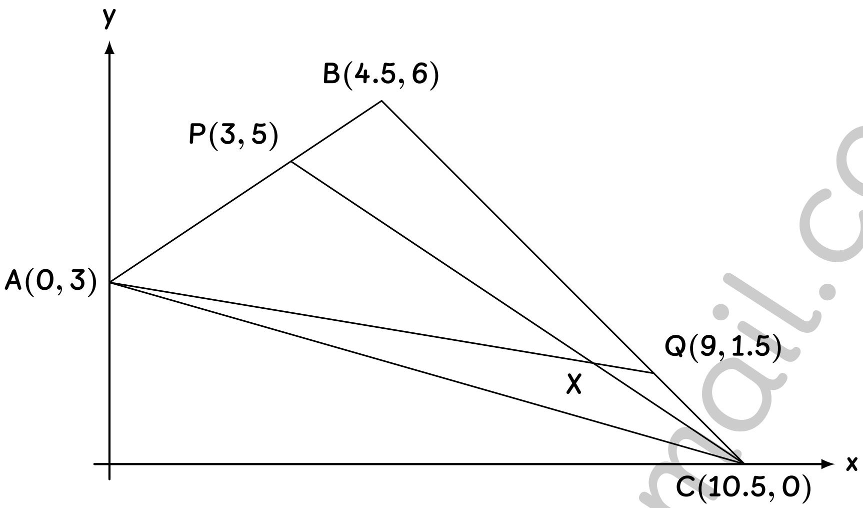
$$10y = -10 \qquad \text{subtracting the two equations}$$

$$\implies y = -1 \checkmark A_1$$

Problem 44

KCSE2022/P2/No. 20

In the diagram below, the vertices of triangle ABC are A(0, 3), B(4.5, 6) and C(10.5, 0). Points P(3, 5) and Q(9, 1.5) lie on lines AB and BC respectively.



(a) Find:

(i) AQ (1 mark)

SOLUTION:

$$\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = -\overrightarrow{OA} + \overrightarrow{OQ}$$
$$= -\begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 9 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 9 \\ -1.5 \end{pmatrix} \checkmark B_1$$

(ii) CP (1 mark)

SOLUTION:

$$\overrightarrow{CP} = \overrightarrow{CO} + \overrightarrow{OP} = -\overrightarrow{OC} + \overrightarrow{OP}$$
$$= -\begin{pmatrix} 10.5 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -7.5 \\ 5 \end{pmatrix} \checkmark B_1$$

(b) Lines AQ and CP intersect at X such that CX = kCP and AX = mAQ where k and m are scalars.

(i) By expressing OX in two different ways, determine the values of k and m. (6 marks)

SOLUTION:

$$\overrightarrow{OX} = \overrightarrow{OC} + \overrightarrow{CX} = \overrightarrow{OC} + k\overrightarrow{CP}$$
$$= \begin{pmatrix} 10.5 \\ 0 \end{pmatrix} + k\begin{pmatrix} -7.5 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} 10.5 - 7.5k \\ 5k \end{pmatrix} \quad \checkmark M_1$$
$$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX} = \overrightarrow{OA} + m\overrightarrow{AQ}$$
$$= \begin{pmatrix} 0 \\ 3 \end{pmatrix} + m\begin{pmatrix} 9 \\ -1.5 \end{pmatrix}$$
$$= \begin{pmatrix} 9m \\ 3 - 1.5m \end{pmatrix} \quad \checkmark M_1$$
$$\begin{pmatrix} 10.5 - 7.5k \\ 5k \end{pmatrix} = \begin{pmatrix} 9m \\ 3 - 1.5m \end{pmatrix} \quad \checkmark M_1$$

$$\implies 10.5 - 7.5k = 9m \implies 9m + 7.5k = 10.5$$

$$5k = 3 - 1.5m \implies 1.5m + 5k = 3 \qquad \times 6$$

$$9m + 30k = 18 \qquad \checkmark M_1$$

$$9m + 7.5k = 10.5$$

$$22.5k = 7.5 \implies k = \frac{7.5}{22.5} = \frac{1}{3} \qquad \checkmark A_1$$

$$1.5m = 5\left(\frac{1}{3}\right) - 3 = \frac{4}{3}$$

$$m = \frac{1}{1.5} \times \frac{4}{3} = \frac{8}{9} \qquad \checkmark B_1$$

(ii) Determine the exact coordinates of point X. (**2 marks**)

SOLUTION:

$$\overrightarrow{OX} = \begin{pmatrix} 10.5 - 7.5\,(1/3) \\ 5\,(1/3) \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 5/3 \end{pmatrix} = \begin{pmatrix} 8 \\ 1^2/3 \end{pmatrix} \qquad \checkmark M_1$$

Hence the coordinates of X are $(8, 1^2/3)$. $\checkmark A_1$

Problem 45 KCSE2022/P2/No. 21

(a) Juma bought a house **4** years ago for **Ksh. 2500000**. The value of the house rose steadily over **4** years to its current value of **Ksh. 3700000**. Calculate, correct to 2 decimal places, the annual rate of appreciation in the value of the house. (**3 marks**)

SOLUTION:

$$2500000\left(1 + \frac{r}{100}\right)^4 = 3700000 \qquad \checkmark M_1$$

$$\left(1 + \frac{r}{100}\right)^4 = \frac{37}{25}$$

$$1 + \frac{r}{100} = \sqrt[4]{\frac{37}{25}} = 1.102974 \qquad \checkmark M_1 \quad \text{use a}$$

$$\frac{r}{100} = 0.102974$$

$$r = 10.2974$$

$$r \approx 10.30\% \qquad \checkmark A_1$$

(b) At the time Juma bought the house in 21(a), Tony also bought a car valued at **Ksh. 5100000**. The value of the car depreciated steadily at a rate of **2**% every **4** months.

Determine correct to the nearest shilling, the current value of the car. (**3 marks**)

SOLUTION:

$$\text{Amount} = 5100000\left(1 - \frac{2}{100}\right)^{12} \qquad \checkmark M_1$$

$=4002055.29$ ✓A_1 use a

$\approx$ **Ksh. 4002055** ✓B_1

(c) The house bought in 21(a) continued to appreciate in value at the same rate while the car bought in 21(b) continued to depreciate in value at the same rate. Determine the number of years from the time of purchase, it would take for the value of the house and that of the car to be equal. Give the answer correct to 1 decimal place.(**4 marks**)

SOLUTION:

Let **t** = time in years it takes for the values to be equal.

$$2500000\left(1+\frac{10.3}{100}\right)^{t} = 5100000\left(1-\frac{2}{100}\right)^{3t}$$ ✓M_1 form an equation

$$2500000\,(1.103)^{t} = 5100000\,(0.98)^{3t}$$

$$\frac{1.103^{t}}{0.98^{3t}} = \left(\frac{1.103}{0.98^{3}}\right)^{t} = \frac{51}{25}$$ ✓M_1

$$t\log\left(\frac{1.103}{0.98^{3}}\right) = \log\frac{51}{25}$$

$$t = \frac{\log\frac{51}{25}}{\log\left(\frac{1.103}{0.98^{3}}\right)}$$ ✓M_1

$$=4.4940837$$ use a

$$\approx 4.5 \text{ years}$$ ✓A_1

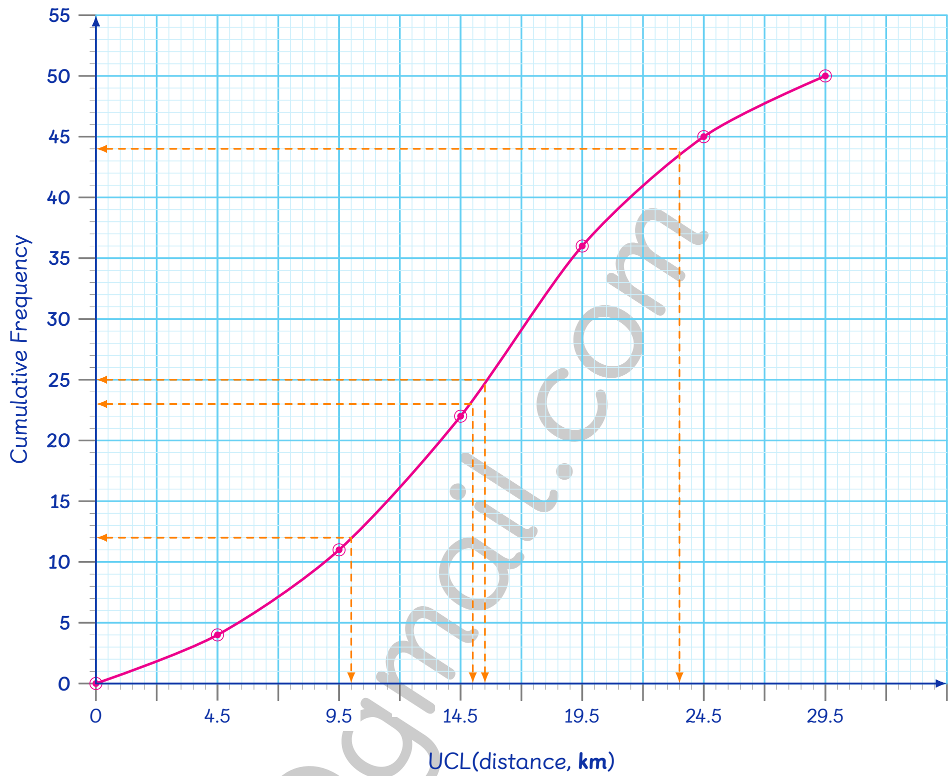
Problem 46 KCSE2022/P2/No. 22

Fifty teachers in a sub county attended a workshop. The table below shows the distribution of the distances (**d**) in kilometres travelled by the teachers from their respective school to the training venue.

Distance **d**(**km**)	0 − 4	5 − 9	10 − 14	15 − 19	20 − 24	25 − 29
No. of teachers	4	7	11	14	9	5
Cumulative frequency	4	11	22	36	45	50

(a) On the grid provided, draw a cumulative frequency graph to represent the information above. (**4 marks**)

OGIVE MARKING RUBRIC	
Appropriate scale used	✓S_1
Plotting of all points correctly	✓P_2
Draw a smooth ogive	✓C_1



(b) Use the graph to estimate:

(i) the median distance. (**1 mark**)

SOLUTION:

median $= 15.5$ ✓ B_1 read from the graph UCL value at $cf = 25$

(ii) the number of teachers who travelled a distance **dkm** where $15 \leq d \leq 23$. (**3 marks**)

SOLUTION:

teachers travelled at most $d = 15$km: $= 23$ ✓ B_1 read from the graph

teachers travelled at most $d = 23$km: $= 44$ ✓ B_1

teachers travelled $15 \leq d \leq 23 = 44 - 23 + 1$

$= 22$ teachers ✓ B_1

(c) Each of the 75% of all the teachers who travelled a distance **dkm** where $d \leq 10$**km**, used a motor bike and each was charged **Ksh. 50**. Determine the total amount of money raised by the motor bike operators. (**2 marks**)

SOLUTION:

teachers travelled at most $d = 10$km: $= 12$ ✓ B_1 read from the gra

teachers travelled by motor bike $= 75\% \times 12 = 9$

Total amount paid $= 9 \times 50 =$ **Ksh. 450** ✓ B_1

Problem 47 KCSE2022/P2/No. 23

In an inter school mathematics contest, schools can register teams in junior and senior categories. Information on number of students and the participation fee per team in each category is given in the table below.

	Junior category	Senior category
No. of students per team	6	4
Participating fees per team	**Ksh. 2000**	**Ksh. 3000**

The organising committee projected to register **x** junior teams and **y** senior teams.

(a) For the contest to take place, the following conditions must be satisfied:

(i) At least two junior teams must be registered.

(ii) The number of senior teams must be more than half the number of junior teams.

(iii) The total number of participating students from the two categories must not exceed **48**.

(iv) The total amount of money raised from the participation fees must be more than **Ksh. 12, 000**.

Write down inequalities in **x** and **y** that satisfy the conditions. (**4 marks**)

SOLUTION:

$x \geq 2$ ✓B_1 at least two junior teams must be registered

$y > \frac{1}{2}x$ ✓B_1 senior teams must be more than half the junior teams

$6x + 4y \leq 48$ total number of participants is less than 48

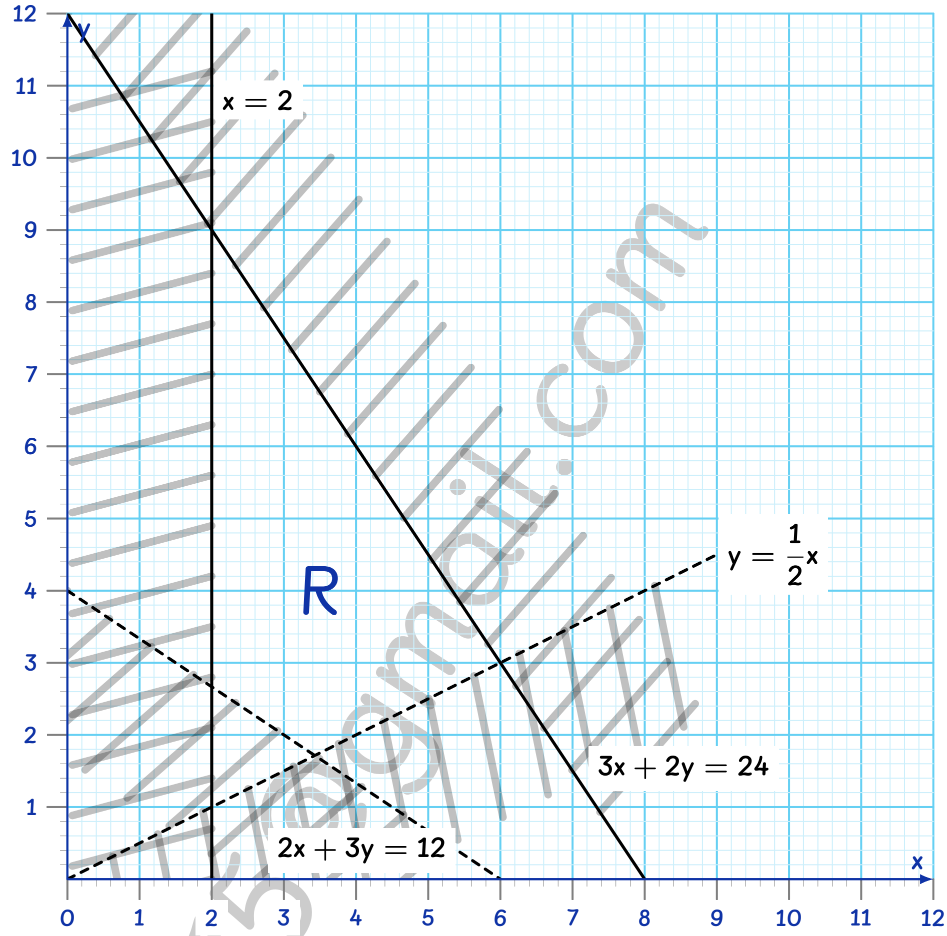
$\implies 3x + 2y \leq 24$ ✓B_1

$2000x + 3000y > 12000$ participation fee must be more than **Ksh. 12000**

$\implies 2x + 3y > 12$ ✓B_1

(b) Represent the inequalities in (a) on the grid provided. (**4 marks**)

GRAPH MARKING RUBRIC	
Plot $x = 2, y = \frac{1}{2}x$	
Plot $3x + 2y = 24, 2x + 3y = 12$	
$x \geq 2$,shade left of the line	✓B_1
$y > \frac{1}{2}x$,shade below the line	✓B_1
$3x + 2y \leq 24$,shade above the curve	✓B_1
$2x + 3y > 12$,shade below the line	✓B_1



(c) The organising committee expected to make a profit of **Ksh. 200** for every junior team and **Ksh. 500** for every senior team that participated. Determine the number of teams each category that should be registered in order to maximise the profit. (**2 marks**)

SOLUTION:

The point (x, y) needed must lie on the corner points as indicated in the required region, R in the graph. These are $(2, 9)$, $(6, 3)$, $(2, 2.6)$ and $(3.2, 1.8)$.

(Note that required point (x, y) must be an integer.)

The best value is at $(2, 9)$

Thus, junior teams, $x = 2$ ✓ B_1 must be an integer

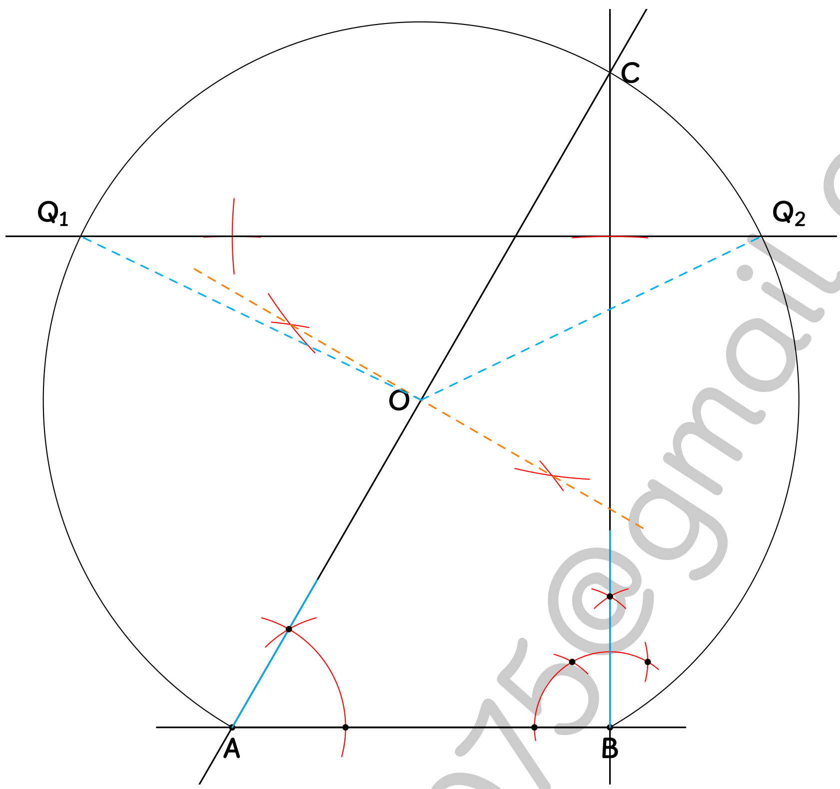
senior teams, $y = 9$ ✓ B_1 must be an integer

Problem 48

KCSE2022/P2/No. 24

In this question use a ruler and a pair of compasses.

The line **AB** draw below is a side of triangle **ABC** in which $\angle ABC = 90°$ and $\angle BAC = 60°$.



LOCI MARKING RUBRIC

Construct 60° at A	✓B_1
Construct 90° at B	
Locate point C and complete the triangle	✓B_1
Bisect line AC	✓B_1
Draw arc centre O from B to A	✓B_1
Divide line BC into 4 parts.	✓B_1
Draw line parallel to AB through 3rd point	
Locate points Q_1 and Q_2	✓B_1
intersection of line and arc	

(a) Complete triangle **ABC** (**2 marks**)

(b) Construct the locus of points **P** such that $\angle ABC = 30°$. (**2 marks**)

(c) Locate by construction points Q_1 and Q_2 which satisfy the conditions below.

(i) Q_1 and Q_2 lie on the same side of line **AB** as **C**.

(ii) Area of $\triangle AQ_1B$ = Area of $\triangle AQ_2B = \frac{3}{4}$ Area of $\triangle ABC$.

(iii) $\angle AQ_1B = \angle AQ_2B = 30°$.

Measure the length of line Q_1Q_2. (**3 marks**)

WARNING(Assumption)

We have assumed:

(b) Construct the locus of points **P** such that $\angle APB = 30°$.

SOLUTION:

$Q_1Q_2 = 9.0\text{cm}$ ✓B_1 $\pm 0.1\text{cm}$

(d) Calculate the area above the line Q_1Q_2 bounded by the locus of points **P**. (**3 marks**)

SOLUTION:

Area of segment = Area of sector OQ_1Q_2 − Area of $\triangle OQ_1Q_2$

$$= \frac{129}{360} \times \pi \times 5 \times 5 - \frac{1}{2} \times 5 \times 5\sin 129°$$ ✓M_2 measure $\angle Q_1OQ_2$ in the diagram

$$= 28.14 - 9.714$$

$$= 18.416\text{cm}^2$$ ✓A_1